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Creativity as an Emergent Property of Complex Educational System

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Abstract
The importance of creativity in education has been discussed often in the literature. While there remains no agreed-upon definition of creativity, the psychological literature points to traits of a creative person. These include the ability to think outside the box, make connections between seemingly disparate ideas, and question norms. The literature provides several examples of classroom experiments to help foster creativity in the classroom. In science and mathematics, we can start by getting students to recognize mathematics and the sciences as being creative endeavors. While these attempts are noteworthy, they are not necessarily aligned with instructional practices. In this article, we propose that to promote creative thinking in our classrooms, we need to see our educational system as a complex system or a network of connections between different disciplines. The 20th century notion that school and college education is rooted in discipline-based reductionism and that learning leads to specialization caters to a few, leaving a large number of students to fail out of the system. The American liberal arts educational model prides itself on giving students a holistic perspective by exposing them to various disciplines. However, merely exposing students to different ideas without having them realize the deep, underlying connections is like expecting interesting dynamics in a collection of disconnected nodes. We propose that the education system is a complex system composed of various nodes, representing different disciplines with the edges representing the flow of unifying ideas between them. Connections between the nodes allow for flow in these paths, resulting in greater opportunity for creativity, which is an emergent property of such a network. The abstract notions discussed above are illustrated by deliberate attempts (ambitious though small) made at the authors' institution to build an educational experience focused on creativity.
Keywords: Creativity, Emergence, Complexity

1. An Introduction to Creativity

Reform efforts in undergraduate science education have addressed the need to instill in students not just an understanding of the content but also an appreciation for the spirit of scientific inquiry (AAAS, 1993; NRC, 2000; Singer, Hilton & Schweingruber, 2006). There is also a need in STEM education to simulate actual

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scientific practices (Handelsman et al., 2004; NRC, 2000), prepare students to be "innovation ready" (Wagner, 2012) and shift learning away from the acquisition of facts and procedural knowledge and to environments that encourage innovation (Handelsman et al., 2004; Southwick, 2012). This runs parallel to the need to cultivate adaptive expertise in our students whereby they are exposed to opportunities to be flexible and adaptable in problem-solving situations (Cropley, 2015; Hatano & Inagaki, 1986), and to deemphasize routine expertise—those most often stressed in STEM education (DeHaan, 2009).

Innovation, adaptive expertise, and problem solving are all associated with both science and creativity. Teaching for creativity, therefore, can be one way to encourage students to do the work of scientists and to prepare them with skills and habits of mind that are necessary for STEM-related careers. However, creativity and imagination are seldom emphasized in STEM teaching and learning (NRC, 2011) with rote and dry instructional practices often leading to students dropping out of STEM fields (Goldberg, 2008). By and large, students, especially in introductory courses, are taught by traditional lecture and their laboratory experiments are usually predetermined, in effect suppressing any chance of creative thinking. In order to encourage creative thinking, creativity has to be cultivated in the classroom and rewarded (Kaufman and Sternberg, (2007). This paper is devoted to a discussion of the theory and practice of creativity in the mathematics and science classroom and the authors' attempts to highlight creativity in the teaching and learning process that occurs in their own classes.

Our instinctive view of creativity appears to come from the image we have of the 'genius'. The American physicist David Bohm (Bohm and Nichol, 2004) stated that creativity is "...very hard to define or specify...it will be best to hint at it." According to them, "one prerequisite for originality is clearly that a person shall not be inclined to impose his preconceptions on the fact as he sees it. Rather, he must be able to learn something new, even if this means that the ideas and notions that are near and dear are overturned." This idea runs parallel to those of Thomas Kuhn (2012), who saw true advances and understanding in science as occurring periodically as major paradigm shifts. It is suggested that creative individuals are easily prone to abandoning preconceived ideas and "defying the crowd" (Sternberg and Lubart, 1996; 1999), leading to discoveries. This view of creativity is not uncommon in the sciences; some scientists believe that "…in the whole of human history, perhaps only a few people have achieved it [creativity]" (Bohm and Nichol, 2004).

Unfortunately, this restrictive view of creativity is also shared by students. A survey of over 250 students conducted by the authors about the creative nature of science showed that undergraduate science and mathematics students at our institution perceived the artistic disciplines to be far more creative than the sciences with math and physics ranking fairly low on the list (Munakata and

Vaidya, 2012). However, students and even scientists are generally unaware of a second, extended definition which claims that creativity can be "less novel and forward-increment current ideas" (Sternberg, 1988; Sternberg, 1999). Such work is not "paradigm shifting" but "adapts within existing paradigms" (Sternberg, 2003). In fact, Sternberg suggests that perhaps the work of Mozart should be considered as an example under this definition of creativity.

The literature on creativity is vast and an exact definition is difficult to provide (Csikszentmihalyi, 1988; DeBono, 2017; Guilford, 1958; Torrance, 1966). Sternberg and Lubart (1991) synthesized these works and provided the key characteristics of creativity which include the ability to (i) connect ideas, (ii) see similarities and differences, (iii) be flexibile, (iv) be unorthodox, (v) be motivated, (vi) be inquisitive and (vii) question norms. In light of these evolving views about creative thinking, there is growing consesus and movement towards cultivating creativity in the classroom (Claxton, Edwards and Scale-Constantinou, 2006; Hamza and Griffith, 2006; Sternberg, 2003; Torrance, 1977). A deeper understanding of the nature (and types) of 'creativity' would help us contribute to the ongoing efforts to revamp math and science instruction in a manner that fosters creativity (e.g., Silver, 1997; Mann, 2006; Bolden, 2012; Leikin, 2009). As noted above, the importance of such efforts cannot be underestimated; in order for students to engage fully in science, we must design classroom experiences that encourage creativity.

The plan of this paper is to draw comparisons between our model of college education and complexity theory. In representing education as a complex system, we are immediately led to defining parallel ideas such as 'connectivity' and 'emergence' with inter- and trans-disciplinary models of learning. Section 2 of this paper is focused on making these analogies; we do so by means of a 'lego-model' of creativity, which is explained in detail in section 2.2. Section 3 takes up the idea of complexity and emergence and how they may be understood in the context of education. Section 4 provides concrete examples of how the authors have attempted to implement the ideas behind these theories in their own classrooms and some of the outcomes of these classroom experiments. We end in section 5 with a discussion.

2. Nature of Understanding

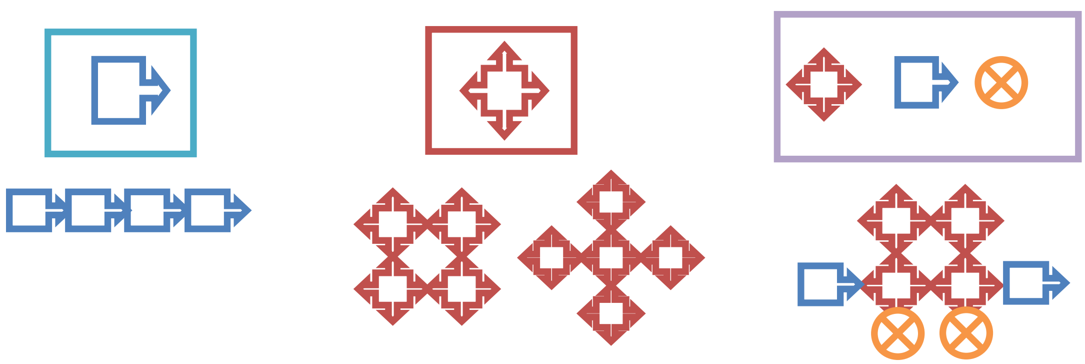
2.1 Learning with understanding

Learning with understanding has been a major goal of science and mathematics educators and is also part of the national goals advocated by NCTM (Hiebert, 1999) and NRC (2002). Hiebert and Carpenter (1992, pg. 68) define the term 'understanding' in the following manner: "A mathematical idea or procedure or fact is understood if its mental representation is part of a network of representations. The degree of understanding is understood by the number and strength of the connections. A mathematical idea, procedure or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections." Perkins (1988) states that "understanding", which is the essential purpose of education, involves getting students to see a concept not just in isolation, but also within its "web of associations that give it meaning". Marshall (2014) state that "understanding" involves seeing the "complexity of something…how it affects other things, and how it is part of a system". Educators and education scholars across time highlight the importance of 'connections' in developing understanding of a discipline and the diversity of meanings that education should allow for (Marshal and D'Adamo, 2011.). The recurring themes of connections and network appear to suggest that education can be viewed and treated as a complex system.

The roots of the theory of complex systems are hard to point to, although one can identify similarities to systems theory of Bertlanaffy (1968) and Cybernetic theory (Wiener, 1961). In elementary terms, a complexity refers to the ways that "open" systems and their environments interact with each other (Koopmans, 2017). In recent decades there is much written about the subject of complex systems and its relationship to education. However, more often than not, the aspects of complexity discussed in the literature have concerned themselves with the logistics and structure of educational institutions as systems and the knowledge flow web (see for instance Morrison, 2006, 2010; Kuhn, 2008; Mason, 2008). For curricular aspects of complexity theory in education, we refer the readers to the pioneering works of William Doll (2008, 2015). In the ensuing discussion, we will focus primarily on the curricular aspects of education, in particular focusing our attention on early college mathematics and science.

The central tenet of the liberal education movement is captured simply in the statement: "A liberal education is about gaining the power and the wisdom, the generosity and the freedom to connect" (Cronnon, 1998). The American Association of Colleges and Universities (AACU) describes liberal education as "An approach to college learning that empowers individuals and prepares them to deal with complexity, diversity, and change. This approach emphasizes broad knowledge of the wider world (e.g., science, culture, and society) as well as in-depth achievement in a specific field of interest" ("What is a liberal education," 2018). A typical US undergraduate college curriculum, composed of a total of 120 credits (about 40 courses) is distributed among core courses, major and minor courses and free electives, each occupying about a third of the total, with variations within disciplines and institutions. Opportunities to 'connect' and cross disciplinary boundaries usually occur in the beginning core courses or highly advanced free electives but are few and far between. There is also much choice provided to the student about how many and even if they wish to take courses that allow for connections. Therefore, there is a disconnect between our professed educational needs and curricular design. Before offering strategies to remedy this situation, it is crucial that we first understand the kinds of connections that can exist.

The terms 'interdisciplinary', 'multidisciplinary' and 'transdisciplinary' have become commonplace in the academy and often used improperly, without distinction (Lawrence, 2010). Of these, multidisciplinary teaching refers to instruction where diverse parallel viewpoints with different goals and objectives are presented in the same course. However, in an interdisciplinary or transdiscplinary setting, goals overlap or even unify completely to one (Park and Son, 2010). In such a case, the students get to see the commonalities between different viewpoints (i.e. disciplines) thereby allowing students to make new meanings out of old ideas. Even between inter- and trans-disciplinary notions, the latter allows for greater "cross-fertilization of knowledge and experiences from diverse groups of people" (Lawrence, 2010). In such a case, the attempt is not merely to seek what lies at the intersection of different disciplines but to blend the two disciplines into a single one, allowing for "innovative goals" and "enriched *understanding*".



a) b) c)

Figure 1. A Lego-model of creativity can be used to depict how an increase in the opportunities to connect to other ideas and diversity of meanings can enrich the kinds of structures that can emerge.

2.2 A Lego-model of complexity and creativity

We propose a *Lego-model* of complexity and creativity which we believe is an appropriate way to distinguish the different ways that learning can be enriched through making connections. Figure 1 captures the essence of this idea. Each panel in the figure above represents a particular mode of learning; the top of the figure shows a toolkit containing Lego pieces of certain kinds and bottom half of the figure shows one or more sample models that can be constructed using such a kit. Each panel represents a course with the learning mode used. Panel (a) is a disciplinary model, where each piece is similar in its make-up, with minimal number of connections, allowing for linear growth. Panel (b) shows a more complex setup where each lego piece has greater number of connections when compared with the very simple units in (a). Such a model of learning can be realized by the presence of the right kind of teacher. When compared to panel (a), the emergent structures in panel (b) are clearly more diverse, complex and interesting. However, even in this case the outcomes are predictable (linear) and novelty simply amounts to scaling. Finally, panel (c) shows the toolkit with a diverse collection of ideas. This model allows not only for a multiplicity of structures but also for ‘original’ or ‘creative’ ones that subsume the former and also far exceed the complexity of anything that could be produced from previous models. The more connections on a Lego piece, the more we allow for diverse interpretations of any concept and allow for more connections. The goals also need to be adaptable so pieces from different boxes do not have static outcomes but can be modified depending on changing goals (making allowance for re-interpretation).. In the language of the lego-model, a critical component of creative emergence is the size of the box; the greater the number of pieces in the box, the greater the possibilities.

A standard measure of creativity, the Torrance Test (Torrance, 1998), uses four scales to measure the level of creativity of a student. These include (i) fluency, (ii) flexibility, (iii) originality and (iv) elaboration. A typical question that is posed on this test might look like the one indicated in figure 2, where one is asked to sketch as many objects as possible using the same ‘incomplete’ shape. In the context of mathematics, one could envision asking the student to consider various interpretations of an equation. The Guilford – Alternative Assessment Test (Guilford, 1968) also seeks to identify the same traits by asking to describe possible alternative uses of some well known object (such as a shoe, for instance). The traits are easily understood from the Lego-model perspective. Having more pieces of Lego in your box can be considered to be akin to having greater fluency. The greater the variety of Legos in your toolbox, the greater is the possibility for variations (flexibility) and room for newer ideas to emerge (originality). All in all, increased number of pieces of Lego and greater the types of Lego pieces, the more opportunity one has for elaboration and originality.

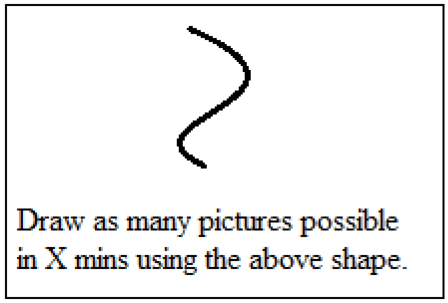


Figure 2. A typical non-verbal question used to measure creativity.

3. Complexity and Emergence

The science of complexity speaks of a world of connections where the system cannot be merely considered a linear sum of individual parts "but the product of the parts and their interactions" (Davis & Simmit, 2003). The post-modern view of nature is non-linear where simplistic causal principles no longer determine the fate of the system. Taking a cue from (post) modern physics, Newtonian notions of linear, force-induced action is insufficient to explain most complex, out-of-equilibrium phenomena in nature. Simple tweaking of the age-old ideas of Euclid and Newtonian in the nineteenth and twentieth centuries have resulted in new areas of physics such as Quantum mechanics and Relativity, which, though linear, have revealed a more spatially complex universe in which the Newtonian world remains a mere projection. Similarly, non-equilibrium thermodynamics provide an alternative, dynamic perspective of our evolving world, which deals effectively with 'time' and 'irreversibility' and allows for dynamic, bifurcations to multiple stable or meta-stable equilibria, i.e. states which are intrinsically tied to the environment and capable of changing to new states as external conditions change (Doll, 2015).

The bifurcations being referred to here cross-cut scales and result from the simultaneous interactions of several components. At a smaller, 'micro-scale' level, students need to be taught concepts which transcend disciplinary definitions. For instance a biographical essay in an English class can encourage students to more deeply learn and speak about history, mathematics etc.; a civil engineering course can get students to ponder more deeply about sociological and environmental issues; students in a computer science class could connect to anthropology and philosophy by trying to understand the history or language and issues surrounding automation, respectively and physics students could engage in conversations about the biology and psychology of hearing (sound) and seeing (optics). While several faculties do engage in such practices within their classes, it is sporadic and often superficially done; only a systematic, university wide adoption of such pedagogical methods will be effective. To genuinely bring about such reform in the learning experience requires changes at a 'meso-scale', i.e. introductory classes in particular, must be team-taught by faculty from different disciplines. The courses must be taught in a genuine interdisciplinary fashion with complete engagement of all the faculty and students present and one must be careful to distinguish parallel-discisplinary pedagogy with interdisciplinarity (Park and Son, 2010). 'Macro-scale' investment in this idea requires reforming the learning goals and major curriculum in any discipline and a change in the way one views the nature of 'understanding'. Courses and programs need to be re-designed to allow diverse perspectives in a class and greater overlaps between the courses students take in different majors, even at the advanced levels. Of course, such a proposal requires that textbooks and assignments in classes be appropriately chosen, greater planning among faculty to design curricula and greater flexibility by administration to permit programmatic overlaps at the expense of a fashionable new program. We contend that the collective implementation of such practices across all scales will inevitably lead to new ways of thinking and learning (and even teaching) which are hallmarks of creativity. Creativity cannot emerge from a base state which is completely deterministic. As the lego-model indicates it is an unpredictable state of equilibrium that emerges from the interplay of foundational states which allow for a multiplicity of connections. In the language of complexity theory, we refer to these equilibria as emergent states. Doll (2008) states: "Order emerges from interactions having just the 'right amount' of tension or difference or imbalance among the elements interacting." In the context of education, the emergence of order, we argue, is creativity. Therefore creativity, at its various levels leads to understanding; it is a form of revelation or interpretation of any content, based on the variety of meanings one can attribute to it. Doll (2008) articulates this idea thus:

"Emergence of creativity from complex flow of knowledge – example of Benard convection pattern as an analogy – dissipation or dispersal of knowledge (complex knowledge) results in emergent structures i.e. creativity which in the context of education should be thought of as a unique way to arrange information so as to make new meaning of old ideas."

Similarly, Hiebert and Carpenter (2006, p. 69) associate understanding, and as a result, creativity, to that of a complex network: "Understanding increases as networks grow and as relationships become strengthened with reinforcing experiences and tighter network structuring."

A more direct connection between understanding and creativity can be seen by examination of the various traits of creativity, discussed earlier. In the context of mathematics education, Hiebert and Carpenter (2006, p.68) argue that seeing the "similarities and differences between different representation forms are the basis for relationships that reappear again and again throughout a student's mathematical career. For example, an understanding of the written epsilon definition of limit is presumed to be enriched if it is connected to the picture of an asymptote on a graph. Similarities and differences between alternate representations of the same information are relationships that can stimulate the construction of useful connections at all levels of expertise.

In the words of Davis & Simmit (2003), "complex phenomenon is emergent, meaning that it is composed of and arises in the co-implicated activities of individual agents. In effect, a complex system is not just the sum of its parts, but the product of the parts and their interactions." Therefore, it appears that the primary task of the teacher is to use the blueprint of a complex system to design a curriculum; this sets it up for emergent creativity. In the following section we present a few examples of our experiments (courses and lessons) that were designed with the deliberate intent of fostering creativity among our students. In particular, we will present our classroom, keeping in mind the practice recommended by Brent and Simmit (2003) for the mathematics classroom to serve as an "adaptive and self-organizing complex system".

4. Examples of Creative Thinking in Education

We describe some freshman and sophomore level undergraduate classroom activities designed by the authors in various courses that aim to foster creative thinking. At the outset we must confess that while several aspects of our practice are derived from keeping the basic traits of creativity in mind and on helping students make connections, it is interdisciplinary at best. A transdisciplinary model of education, which might perhaps be best suited to help foster creativity requires deep commitment from the entire university system and a revamping of our current practices which would be harder to achieve, though not impossible. A transdisciplinary model of learning requires the abandonment of the primacy of disciplines in favor of learning goals. At the university level, it requires the university to abandon the reductionist practice of creating new departments and undergraduate programs; faculty need to learn to speak new languages and engage at a far deeper level in the scholarship and teaching with those from 'other disciplines' and students will need to forfeit affiliation to a 'major'. In many ways, this requires a major upheaval of the current structure of a university and the biggest challenge to such a change is our collective mindset. However, one can slowly move towards such a goal through increasing inter-disciplinary practices which are constructed with the goal of increasing the flow of knowledge across departments and where students and faculty learn to truly appreciate that true knowledge about anything can only be achieved by the pursuit of its diverse facets.

4.1 Creative thinking

Two of the authors, Munakata and Vaidya, are part of a university-wide effort to design a course on creativity. This course, *Creative Thinking*, has now been in existence for nearly four years and has been taught by one of the co-authors (Vaidya) multiple times. This course was designed with the very principles discussed in this article, although we were not aware of the connections between creativity and complexity at the time. While the instructor spearheads the course, it is a team effort and taught with the aid of faculty from nearly all disciplines on campus. Students in the course also represent different majors on campus. Since this course is not content-based, it allows for more freedom to explore ideas and connections than a typical discipline-based course where certain concepts and ways of thinking need to be mastered. The curriculum was based on the inter- and trans-disciplinary model, where various disciplines were brought to bear on the discussions with the goal of finding similarities and differences and ways of diversifying the students' toolkits. One example of a lesson in this course would be a discussion on 'space' and 'time'. Over the period of a week, students discussed this notion with a physicist, mathematician, dancer, musician and religion scholar, covering various interpretations which included a discussion about matter, metrics to measure space, 'space' as the allowance of a medium to construct a performance, 'time' as a measure of beats in music, as an indicator of memory, as merely a parameter or as a measure of distance of astronomical bodies in the universe. By realizing the 'affordances' of space and time, students were provided with an opportunity to make new connections. In another class, we discussed the Chladni plate patterns (see figure 3) and tried to interpret it in the language of physics, music, and art. The ensuing discussion led students to consider the possibility that art and science tell similar stories and music and art can be described by the language of 'energy flow'.

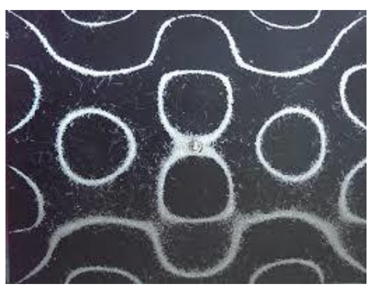


Figure 3. The Chaldni plate patterns were used to motivate discussions about the nature of sound and music, physics and art. This experiment is in itself a pertinent representation of how changes in input conditions (frequency, in this case) give rise to emergent structures of higher complexity.

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In another class, students were asked to visit a typical strip mall near their home and examine the various kinds of stores there. A biology co-instructor then brought up the concept of environmental niches and students immediately saw the connections between biological and social structures. In all of these classes, the faculty was a mere facilitator, providing input when necessary. Students were active participants in exploring the ideas – often in teams, conducting experiments, doing the research, brainstorming and trying to make connections.

4.2 Contemporary mathematics for everyone

In a recent general mathematics course for non-math and science majors, co-taught by all three authors, we made a conscious attempt to infuse creativity into the teaching and learning of mathematics. Students in this course, representing a variety of majors, experienced mathematics as a process of discovery, through construction rather than passive lecturing. The instructors' objectives were threefold: (i) hook the student through play, (ii) surprise the student by selecting appropriate problems and exercises which would elicit the much sought after "Aha!" moment, (iii) showcase mathematics through the eyes of the students – not the instructor – by de-emphasizing mathematical thinking as a singular process with absolute answers. Students were encouraged to personalize the process and find their own meaning (Monahan, Munakata and Vaidya, in press). Topics covered in this class included Euclidean and non-Euclidean geometry, probability, data and its visual representation and symmetry. As noted earlier, this course proved to be a little more challenging since the course involved specific content, but the mathematical topics we explored were amenable to these adaptations.

Many of the classroom experiments were selected by instructors so that the unsuspecting student would have no clue that what they were actually doing was solving a math problem; this was only uncovered upon reflection (Monahan, Munakata and Vaidya, in press). One example of this was an experiment students completed outside where they splashed droplets of water on a premade poster. At the time, students were unaware that the collection of their posters created a world atlas. Students made predictions and had a whole-class discuss the possible meaning of their splashing experiment through which they eventually came to the conclusion that this exercise yielded some information about the relative areas of water to land on the Earth's surface through probabilistic analysis. One of the tests in this course required students to analyze data they were given but also collectively synthesize the different data sets given to a larger group and make collective sense of it all. Reading assignments and journal assignments required students to think about connecting to historical developments in mathematics and their own discovery process, finding connections between simple and complex mathematical themes to their personal lives, current and future.

4.3 Art of science project

As part of our efforts to improve physics teaching at our institution, one of the authors (Vaidya) implemented new strategies in some of the courses he teaches on a regular basis. Specifically, over the past three years, he has been using his investigations on creativity to inform the instruction of his Classical Mechanics (PHYS-210) course. This course has been taught using the project-based learning model and is centered around a final project, inspired by questions posed by artists. In one iteration of the course, students worked on building a bicycle powered generator which was utilized in an artistic production (Leszczynski et al, 2017) while in a newer version of this course, students worked on building a Windwalker, also referred to as a "Strandbeest", the brainchild of the well-known kinetic artist Theo Jansen ((http://www.strandbeest.com/). The development of the bike-generator was part of a bigger project called the *Art of Making Sustainable Science* funded by the American Physical Society, where students from physics, mathematics, english, communication and theater worked alongside faculty members and a visiting artist to create a peformance and short films (see https://handspuncinema.wordpress.com/2013/05/01/diane-sawyer-live-an-exclusive-interview-with-miss-piggy/). Details about this course and the positive impact on students engagement with the subject has encourgaed us to continue this approach (Munakata and Vaidya, 2015). In another section of the course, students worked on building the windwalker using simple supplies such as straws and cardboard. In certain lab sections allocated for this project throughout the semester, students brainstormed, planned and developed versions of the windwalker, with minimal guidance from the instructor. At the same time, students were asked to be mindful of the theoretical elements of the course which could inform the physics of this artefact (such as the inverse pendulum model). In subsequent years, students have used 3D printing technology to design and print the windwalkers. The goal of the instructor was to get students in the class to mimic the work of scientists as they go through a discovery process. Students were encouraged to view and rearticulate problems they had to solve in various ways (i.e. connect to the problem in different ways) without forcing a particular solution. Amazingly, students were found to completely abandon a half-solved problem and seek alternative paths as they managed to familiarize themselves better with the subject. The class sought and were directed to various sources from art and science to resolve problems that they encountered.

4.4 Outcomes and student response

Overall, the response of students to the attempts described above has been very positive. In fact, both positive and "negative" comments by students appear to evoke the kinds of reactions that we expect and hope to see such as excitement and enjoyment of the process, a sense of reduced stress in the learning process, an appreciation of learning for learning's sake or fear of the open-ended process where the lesson-plan is not always prescribed but develops slowly with the acquisition of more knowledge. Evaluation of the courses was conducted using focus group interviews and the tests of creativity (pre and post), as well as surveys on students' perceptions about mathematics. Students in all of these courses reacted to the course showing an awareness and appreciation of creative problem solving, diversity of ways of thinking, acceptance of failure and comfort with open-endedness (Munakata and Vaidya, 2015; Leszczynski, et al, 2017; Monahan, Munakata, and Vaidya, in press).

Students commented on their experiences with the course and connections they saw between mathematics, creativity, and topics outside of the classroom. One student noted that, "Through the final experience, I fully realized the connections between mathematics and creativity and my future. While working on the final journal prompt, I tried thinking about just how it all works with public relations. I figured that it takes critical thinking to deal with the public but I didn't realize until I started the final experience that there are many more factors in play with that". Another student noted, "There were many interesting topics in class being discussed today that helped make class feel less like math and more like sociology/health/history class" further identifying connections between mathematics and other subjects. One student stated: "Personally, I love architecture designing and computer graphics, and because of the clearing (up) of learning geometry, and all the inter-connecting math topics we learned, I can hopefully easily understand how to precisely trace graphics, morph objects, and create detailed symmetrical/abstract shapes."

Interviews and surveys can only reveal the student's disposition or change in attitudes to the subject. True creative abilities are harder to evaluate. The Torrance and Guilford tests of creativity were also administered to students in several of these classes (MATH 106, CRTH 151) and showed incremental and positive change. The Torrance test evaluates students' ability to create interesting and distinctive pictures when given a stimulus in a series of three, 10-minute sections. Similarly, in evaluating students' creativity, Guildford's Alternate Uses Assessment provides students with common, everyday objects and asks for up to six other purposes for the object. The results of the pre and post Torrance and Guilford tests revealed significant improvement in 'fluency' and 'elaboration' with t-test scores of 2.6, 2.7 and p-values 0.014 and 0.018, respectively. No significant change was observed in 'originality'. Similarly, the Guilford test also showed significant gains with t and p-values of 2.63 and 0.013 respectively.

No matter what the metrics show us at this early stage of our research, we would note that 'creativity' is not a skill or talent that can be developed through classroom intervention alone. Time is a crucial factor and so is the student's willingness and ability to adopt the practices that promote creativity while keeping at bay poor learning habits that one typically adopts over the years. Longitudinal studies are required to better evaluate our courses and consider their modifications. At this stage we use the student responses to our questions as a gauge of their evolved mindset and perception, which are necessary precursors to deep learning.

5. Discussion

Davis and Simmit (2003) posit that setting up a mathematics classroom as a complex system requires the combination of five conditions: (i) internal diversity, (ii) redundancy, (iii) decentralized control, (iv) organized randomness and (v) neighbor interactions. These are essential structural requirements which provide the conditions for creativity and are also related to the metrics used by Torrance (1998) to gauge creativity. We use this framework to reflect on our experiences with the development and implementation of the activities described above.

For instance, condition (i) is merely a demand for disciplinary expertise or fluency as represented in the Lego-model by each piece (figure 1). While this point has not been discussed in detail in this article, it goes without saying that higher forms of creativity can only occur when there is sufficient background knowledge. To a large extent, creativity can therefore be thought of as a post-nucleation event, i.e. happening after the acquisition of a certain critical mass of information. This can also help explain the conflicting views of creativity. When content knowledge is high, the resulting emergent outcome is *Creativity* while in the case when content knowledge is still forming, we can refer to the emergence as *creativity*. In fact, one could argue that there is a spectrum of creativities whose 'magnitude' is proportional to knowledge content. Hence breadth without depth would be looked upon as limited in its creativity (except perhaps in the case of children) and depth without breadth, without connections, would leave little opportunity for creativity; one needs both! Secondly, internal diversity can be seen necessary to foster flexibility. Diversity allows for greater number of detours, i.e. possibilities to solve a problem. It seems clear therefore that scholars of creativity and complexity are often speaking of similar things.

As we reflect on our own attempts to infuse creativity in our classes and other activities, we note that we were successful in meeting some of the conditions of setting up a complex system suggested by Davis and Simmit (2003).

The courses and activities we discuss above were diverse in terms of student backgrounds and majors and had a loose structure to it which allowed cautious meandering depending upon the interests of the class. Even in an upper level class, where all students had the same major, diversity of thinking was encouraged as groups drew upon individual expertise, and was seen to naturally come through in group discussions. Having said this, while all of the students in the advanced course had taken all of the pre-requisite courses, some had forgotten the basics or had not taken other courses which could have informed their thinking. The diversity that was evident was more in the level of preparation.

The course also exemplified decentralized control, another condition necessary for a complex system. Several sessions in these classes were student-led with the instructor serving as a facilitator and guide. In the final projects conducted in PHYS-210, the instructor simply posed the question and offered no solution. In several cases the 'instructor' had to study and figure out the solution with the student. Collaboration, discussion, experimentation and falsification[2] (Popper, 2005) were part and parcel of the courses. Students were asked to explore freely without regard to failure. While complexity itself was not on the forefront of our minds, the attempt was to get the students to mimic the behavior of a practicing scientist or mathematician, albeit at a less rigorous level. A more detailed discussion about these classroom experiments can be found in previous reports (Munakata and Vaidya, 2015; Leszczynski, et al, 2017; Monahan, Munakata and Vaidya, in press).

6. Conclusion

Creative discoveries in science and mathematics are not just a function of our education system but depend greatly upon the mindset of the scientist. Those with greater proclivity and appreciation for all forms of knowledge set themselves up for more creative discoveries. So, while modern day specialization demands the need for disciplinary knowledge, all aspects of our modern civilization outside the university beg for more creative citizenry. If the academy is to play a role in this endeavor, it must help provide a glimpse of the 'forest' as well as the 'trees'. A coupling of various forms of knowledge where each can influence the other is needed. This requires that fields such as math and physics also evolve along with other disciplines. Only then will such a co-evolution of ideas result in the emergence of new ideas and ways of thinking.

[2] By falsification, we refer to the notion introduced by Karl Popper who stated that science progresses not by assertions and verifications alone but primarily by falsifications of outdated and incorrect hypothesis.

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