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Academic Skill Learning and the Problem of Complexity I: Creational Purposeful Integrated Capability at Skill (CPICS)

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Abstract
Physical and mental skills are intended to achieve success at acting purposefully. As capability at any skill increases, the need to adjust details of application to complexity of context and goals will increase as well. It will become more and more important to prepare mentally for what I now term *Creational Purposeful Integrated Capability at Skill (CPICS)***.** This paper develops what I mean by CPICS. Theory concerning Complex Dynamical Systems (CDS) such as the brain and other evidence points to the likelihood that the mental operations by which our brain produces any kind of skillful behavior cannot remain constant, but rather must develop through stages for skill to progress most profitably. Using early stages of math learning as an example, I propose that what can hold back some students at development of a skill is that even if presented with all the information need for progress, some students have not yet discovered how to make the most useful mental restructuring that is also needed. This paper proposes and discusses as an example details of what may be especially useful restructuring for early stages of math skill learning. This example is then taken as helping to identify the more general type of restructuring that is especially useful for addressing complexity of application that produces CPICS at every stage of skill improvement.

1. The Role of Mental Restructuring in Skill Improvement

The discussion that follows builds upon earlier work (Gardiner et al, 1996; Gardiner, 2000, 2003, 2008, 2019). By skillful *"engagement"* (Gardiner, 2008) I refer to the specific brain actions that produce skillful physical behavior (such as at walking) or skillful mental behavior (such as at solving a math problem).

William James pointed out more than a century ago (James, 1890, 1896) that to live in a complex world we must simplify our interactions with it. But, to paraphrase Einstein's famous saying, we must think as simply as possible, but not more simply than possible.

2. Insight from Bicycle Riding, Theory of Complex Dynamical Systems (CDS) and Related Evidence

Physical skills such as at learning to ride a bicycle illustrate what this paper now discusses in relation to academic learning as well. Once learners understand the bicycle and what they must accomplish, further progress must depend on their somehow developing better ways to use their brains to produce bike riding skillfully. Suggestions and help at training by parents, and training wheels can help, but ultimately *qualitative improvement in engagement* must take place out of direct control by the learners and outside of their conscious awareness. Capability at riding suddenly *jumps* from not possible to possible. Once possible the capability may continue to develop. But not until this first step.

Development of academic skills such as at math, I now argue, also depends on improvement in brain engagement particular to that skill, though not as visibly initially. Bicycle riders cannot ride at all until they make the qualitative engagement change. The math learner who has not made such change at math thinking can still at first manage to some degree with less adequate engagement, but must work harder mentally to compensate and increasingly all but the strongest can be expected to fall behind. And as with bicycle riding, further engagement changes that further improve capability cannot take place until the first step has been made.

Why must the brain apparently change its operations as it builds skill at bicycle riding, or more generally as I now argue? Our brain's enormous complexity appears to be at the heart of our most advanced capabilities (Chomsky, 1972), and its highly complex operations develop in time and in mental spaces created by the brain, and thus are *dynamic*. General properties of Complex Dynamical Systems (CDS) such as the brain have been under study since the middle of the 20th century. Current work is exploring implications of this theory to Education (Koopmans, 2014; Koopmans & Stamovlasis, 2016). Here we now examine how this theory and related evidence can help us understand why all skill development, including academic skill development, is likely to involve changes in how the brain engages at a kind of skill:

1) *The portion of brain activity devoted to any kind of capability is likely to be isolated to a sufficient degree functionally so that special capability can develop.* A complex system must often develop specialized functions (such as at bike riding or math) distinguished from the operations of the system as a whole, through use of *subsystems* (von Berthalanffy, 1969). The subsystem for a particular skill can be expected to depend on activity not only in one but rather in many parts of the brain. The ways in which different subsystems pull together and manage strategically the resources for different kinds of skill cannot be entirely identical, for the operational goals the subsystems address are not identical, but as discussed in a companion paper and Gardiner (2019), subsystem operations can

become strategically similar in ways that can have important implications for skill development.

2) *A subsystem may itself involve further division into functionally interacting subsystems.* Here we are especially interested in how a subsystem producing a kind of skillful behavior develops engagement capability for execution of skillful actions.

3) *It is likely that for skillful behavior to continue to improve, a subsystem producing any kind of skill must change its operations in stages*. In living creatures (Maturana, 1970; Maturana & Varela, 1973) operations of brain and other systems must be sufficiently stable at any time for the creature to be able to live (see also Wiener, 1948). On the other hand, the human brain continues to grow and develop its capacities significantly after birth. The need to retain stability but also to improve operations over development supports the value of evidence for staging found in overall mental development (e.g. Piaget, 1985; Dawson and Fischer, 1994). Watzlawick, Weakland and Fish (1974) have distinguished two ways for system performance to improve. By first order change they refer to improvements that take place without basic changes in system configuration. But greater improvement can require second order change, where a subsystem reconfigures itself in some way to achieve a new functional capability. Nicolis and Prigogine (1989) in fact propose that a measure of complexity of a system is its capacity to make reorganizing transformations. The importance of staged development in brain systems as a whole supports the likelihood of such staging also in subsystems devoted to kinds of skill. Chase and Simon (1973) provide evidence of such subsystem changes as skill at chess develops. Developmental changes specific to a kind skill can explain movement of capability for a particular kind from more general features of capability (Ackerman, 2011, Ericsson, 2013; Ericsson et al, 2006).

4) *Jumps in Skillful Performance*: Evidence that skillful performance can sometimes jump upwards as skill advances (Zeeman, 1976, Stamovlasis, 2016, Sideridis and Stamovlasis, 2016) implies that some change in functional operation has taken place.

5) *Integration within Subsystem Development*: Systems and subsystems profit from integrated operation, as the actions of a thermostat meant to help control house temperature illustrates. The thermostat affects the house temperature most efficiently through connections that integrate thermostat actions with production of other actions by machines that cool or heat the house. Integration of operation

with application within a brain subsystem producing skillful behavior can be expected to profit from such integration as well.

6) *Capacity for Bifurcation in System or Subsystem Development:* As a complex system develops, it can reach positions where its further development can proceed in different ways. "Bifurcation" refers to a position in development where two different paths for further development become possible (Nicolis & Prigogine, 1989).

3. Mental Strategy Addressing Complexity in Purposeful Application of Skillful Behavior

I now wish to distinguish two general strategies our brain can use to engage purposefully skillfully in real time. We all know examples of these strategies. As we will see, these strategies are typically integrated. But strategically they approach skill in different ways.

1) *Reproductive Execution of Skillful Action*. Here an executed action is intended to reproduce as faithfully as possible action that has been developed previously, and has been found sufficiently useful that capability for its reproduction has been retained. Examples include the basic act of speaking the sound for the English language letter "b", and the larger integration of this with other acts involved in speaking a word such as "bat" once this capability has been learned.

2) *Creational Development of Skillful Action*. Here the executed action is not intended to be fully developed dynamically in real time until execution, execution then adjusting dynamically to specific combination of details of context and need which cannot be anticipated in advance. The actions of driving a car, for example, must be adjusted dynamically in real time as the driver moves down the road.

In practice we typically *integrate both types of strategy* to produce skillful behavior, but in ways that can again differ strategically. Actors who memorize their lines in advance and then try to repeat them as faithfully as possible are using reproductive strategy overall but will still adjust the way they speak their lines with creational strategy depending on how action develops during a performance. In a conversation, on the other hand, what one says is usually not prepared in advance, but rather is developed overall with creational strategy adjusting to what has been said, and what is intended in response. But the acts by which words are spoken to an important extent have been prepared in advance as one learns to talk.

4. Developing the Concept of Creational Productive Integrated Capability at Skill (CPICS)

The elements of CDS and other evidence just reviewed implies the likelihood that for any kind of skill to develop, the way the brain develops that kind of capability must go through stages of restructuring appropriate to the goals of that skill. This has implications for academic skill learning, as we will now examine using early stages of math learning as an example.

5. Math Learning Difficulty

Current well researched, and carefully designed curriculum in mathematics has been in place for almost a decade in many states including Rhode Island and California. Yet in most recent published data, as Rhode Island standardized math testing begins in 3rd grade, less than half of the students (44.2%) met grade level expectations, and this percentage was still lower in students from economically challenged families. These percentages went steadily downward reaching 14.6% overall in 8th grade. California was only a bit better. 49% overall met expectations at end of grade 3, and percentages again dropped steadily downward in higher grades. These numbers could then show that many students are simply not able to learn mathematics at the level now expected of them. This would be very unfortunate, for math skills are very important today. But evidence we now review and its implications suggests that some and perhaps even many students may be held back for other reasons we will now discuss.

6. Development of Skill at Arithmetic

6.1 Learning how to use math

Students at math often show their greatest difficulty not at learning the operations of mathematics but rather at learning how to use them productively beyond the specific illustrations covered in class.

Several years ago I had the opportunity to work with a small group of teenagers learning math. When I asked them to solve an algebra word problem similar to what had been covered in class all of them succeeded. I then asked "Who can tell me a problem that can be solved by addition?" All but a few could answer this. I repeated the question concerning subtraction. Now only about half could answer it. When I got to multiplication and division only one boy in the class even tried to answer. He did so correctly.

Math teaching has often focused on how to do operations of math. Increasingly calculators and computers can do such operations for us. But to use such operations more adequately, one must be able to go beyond the specific details of applications covered in class.

The problem of application I am addressing here can be illustrated by demands of medicine. Medical students spend many years learning fundamental skills and then seeing examples of how these skills can be applied to specific cases. Nevertheless, once they begin to practice medicine they will face the fact that each patient will be unique in his or her complex combination of specific challenges. The skilled doctor must build ability to marshal skills flexibly to address the varieties of challenge the patients present.

6.2 Learning how to apply integer addition and subtraction productively

Part of learning skill at math concerns how to perform its calculations mentally, or on paper, or these days, with a calculator or computer. These procedures can be explained, practiced and memorized, and as a student advances, such information can also be found in books, or, these days, online. But knowing how to do mathematical operations does not guarantee that users also understands adequately how to apply them productively, beyond the specific applications covered in training they can remember and adapt.

Let us now consider some examples of how a teacher may develop learning by beginning math students at how to apply the arithmetic operations of addition and subtraction productively to applications involving Integers.

We will see that there can be significant differences in how the student is taught to engage with these skills, and that this can significantly affect how broadly a learner can come to apply such skill productively when tested, or more generally.

1) *Learning to apply addition and subtraction facts.* We begin with an approach that in total is no longer specified in many current math standards, nor in prior standards from which it was developed. But since factual learning as illustrated here is still widely used within teaching as a whole, and is considered especially important by many parents and others in the population, it may also still have some role as some teachers train early stages of math. Let us look briefly at how it can be applied at beginning stages of arithmetic.

In this approach students learn perhaps through memorizing tables or simple arithmetic equations *factual information* about what addition and subtraction operations accomplish. Thus it is a fact to remember that adding 1 to 1 gives 2 and that subtracting two from 5 gives 3.

To then apply such facts to a problem during a test or for other goals, students must carry out a sequence of mental actions. These include deciding if any of the facts learned so far can be applied usefully, retrieving the needed fact or facts from memory, considering and then developing mental actions that apply it to the problem, and finally developing further mental actions to translate the remembered result from the factual information to answer the problem.

Such multistep process can be time consuming, and thus compromise test performance for some students, and will be most straightforward only when facts as presented and learned match the application well enough. If students who have been taught that 1 plus 1 gives 2 are told Mr. Jones has one cow and adds one more, most children taught this way can say that he now has two. But if the question is formulated slightly differently. e.g. “Mr. Jones has one cow and wants to have two. How many further cows must he buy? “, some children can have difficulty realizing that the same fact can be used to answer that question, unless this has been discussed and demonstrated in class, or the subtraction fact that 2 minus 1 gives 1 has also been learned and its use with such a problem also has been demonstrated and understood.

This training does not promote change in how a student thinks qualitatively about math. Learning how to store facts and rules and application examples and then recall this information when needed basically involves extending reproductive strategy every student has had to begin to develop as they learn to talk before schooling even starts. Students who have become good at remembering facts and rules and are strong at more general creational engagement at reasoning can be expected to do the best at applying this kind of training. It is not surprising that some students struggle with this burden, and become increasingly frustrated by mathematics. Working at math can indeed involve reasoning, and thus can help at the training of reasoning capability more generally. But reasoning at math also profits when a student develops engagement specific to math as we will discuss.

2) *Learning to use a calculator or computer to perform operations of addition and subtraction:* Today some teachers may increasingly develop a variant of training 1 that involves showing students how to perform addition and then subtraction operations on a hand held calculator or computer, then explaining to the students how to connect what is calculated to solving a problem. Thus application by students is no longer limited by factual information they can recall and they can to some extent learn rules about operations by discovery rather than memorization.

As with training 1 this training can also still remain heavily reproductive in strategy: remembering specific operations and examples of how to apply them. But a student who does not understand addition and subtraction adequately conceptually may still have trouble knowing how to connect a given application to a calculation they can perform. “I have 26 tables but need 50 for the dinner next week. How many more must I rent?” is easy to solve only of you realize that

50 – 26 will give you the answer. And there can be many more complicated questions than this. Most students can still relatively easily learn to reproduce application acts the teacher or other students demonstrate, but some can still have difficulty going beyond this.

3) *Building development of operational and conceptual understanding of "Number of"*. Today's curriculum at math usually begins before facts discussed in training 1 or calculator or computer use of training 2 are considered. Extending emphasis in many earlier curricula, it starts by Kindergarten if possible, to develop "number sense", i.e. an understanding what a number can mean.

As proposed by Gelman & Gallistel (1978), development of number sense typically starts with the children learning to say number names and then write number symbols in count order, and then using such counting to count numbers of objects of a given kind in a set of objects. Each number word or symbol is paired with a specific object during the counting, and the final number reached is spoken of as indicating an *amount* of objects that have been counted.

But there is evidence reviewed by Susan Carey (2009) that developing the mathematical meaning of a word such as "four" or its symbol 4 or still more "eleven" or 11 will require children to change ways of thinking about words or symbols they have already developed through talking. This involves qualitative change in engagement such as we have been anticipating. And as will now be discussed, some children may have difficulty in making what is the substantial transition from verbal to mathematical thinking.

According to Carey, our use of mathematics in relation to amount builds upon two conceptual abilities which we have at birth. One core ability allows us to compare or detect change in amount. This core capability shows an evolved interest in use of quantification, but according to Carey must be developed further if precise quantification is needed. The quantification precision of integers is instead built, Carey argues, on a different core capability, initial capability for distinguishing between *small* numbers of objects (up to about 4) *held simultaneously* in short term memory. That is, children can realize immediately, without counting, if there are 1, 2, 3, or 4 objects to which they attend. According to Carey, the vocal or symbol writing acts of counting then begin to take on mathematical meaning when used to distinguish these small changes in quantity that children already understand. Once this initial relationship between counting and quantity is developed it must then be extended to give a precise amount meaning to any number that can be reached by verbal counting. Then conceptually there is no limit to the number of objects for which count has this quantitative meaning.

To understand mathematic meaning of number name or symbol in this way represents a significant change from how children have learned to treat words

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or symbols mentally as the have learned to talk. Nouns classify and refer to and identify objects, verbs classify activities, but the amount meaning of a number word or symbol now takes on mathematical meaning according to *position* in a count sequence representation (Gardiner, 2008b). There is nothing in verbal thinking that refers to *position* in a representation of stored information in this way.

This transformation to a new way of understanding the mathematical meaning of a number word or symbol is the first transformation that separates skill learning at mathematics from verbal skill learning children have begun to develop earlier.

The transition we are discussing (Gardiner, 2008b) does not imply that the entire count sequence itself becomes somehow stored in the brain. More likely, engagement operations in the brain will begin to refer implicitly in some way to sequential properties of the count representation, such as that movement along it proceeds in *steps*, and that the sequence of positions in the sequence is *ordered* from lower to higher, and thus has a specific type of ordered *succession*.

Math curriculum already involves many counting related number sense building activities. These can be expected to become increasingly difficult to perform without the mental transition just discussed. But as we will now see, it is still possible for some children to go quite far at performing counting related operations intended to develop initial number sense without making the necessary mental transition, especially if they are particularly adept at thinking verbally.

For example, they may try to think of a number word "three" in a way that already works for verbal language. Perhaps "three" is a *temporary name* as when children play Jack and Jill and one says "I'll be Jack, you be Jill". The symbol "3" could then also refer to a temporary name. Thinking this way, the child can then still answer "how many did you count to?" by giving the temporary name or related symbol when the count ended. Thinking in this way could in fact help some children to understand why the name or symbol given to a specific object changes when the count proceeds in a different order. Why not? Names are only temporary.

Many number sense questions can then be answered by applying audible counting or even without counting out loud once one has learned to say them internally without speaking. As number sense questions become more challenging, ironically, those who are more advanced verbally (and this is more likely to be girls than boys) may well be the ones that meet this challenge most effectively, and may as a result find it hardest to move to a new way of thinking that is no longer verbal.

How might a teacher help students to make this first transition? One, I suggest, is from the beginning to tell the students that they must learn to *think differently* about numbers than about words, and that she or he will be trying to help the students learn how to do so. Carey's review already implies that development of number sense should most effectively begin with intensive work with small numbers of objects where amount can be most easily understood. It seems likely that many teachers may already be doing these things. Another strategy less likely to be in use to try, I propose, could be to immediately begin to explain by using language that specifies exactly what type of *amount information* is held by the count sequence at this stage of mathematical development. If one uses *"number of"* rather than *"number"* as much as possible, this can encourage some children to move away from thinking of numbers more abstractly, which may confuse some, or as temporary names, and instead towards conceptualization that emphasizes that the sequential count representation at this stage carries information concerning "number of" and that the information concerning "number of" is associated with position in the "number of" count sequence.

To speak of "number of" can also usefully emphasize purposeful meaning of number reached by counting. Piaget (1985) argued that ability to understand abstractions developed at a later stage of brain development. Children, like all of us and still more so, seem especially eager to learn how to do things that they believe will have productive value to them. Once children build connection to "number of" in their brain they can say "I want two of those" or "You have four toy soldiers. Can I hold one of them?" Most children can be expected to value such practical purposeful capability.

I want to emphasize that through development of engagement that is organized around a number-of counting sequence, operations giving meaning to a number (here integer) and its purposeful application become *integrated* in a way that facilitates its purposeful creational use. Saying to a friend "You asked for all eight of those. I can lend you six "would be very difficult and for some even impossible without such new mental representation.

4) *Clarifying connection of arithmetic and subtraction to representation of "Change in Number of":* Once -the purposeful meaning of an integer as representing "number of" is connected mentally to *position* in the number-of sequence, this foundation now prepares for another transition that can add operations of addition and subtraction as involving changes in position on this ordered number-of sequence and interpreting these operations as involving "*change in number of* ", perhaps represented in another sequence.

Addition then will involve movement up, and subtraction down the number-of sequence. A farmer starting with three pigs and then buys three more, moves "number of" from three up to six. When he sells two, this moves "number of "down to 4. Thus the farmer ends with four pigs after these transactions.

Students who have built use of this representation can understand the productive value of these operations immediately. And they can use the operations purposefully in ways they invent. “I have only three marbles. I need two more so that I will have five like Betsey”.

Note the precision of meaning concerning integer arithmetic a child has reached. The information and rules a child must learn by training 1 discussed above are captured implicitly, and the purposeful value of integer operations of addition and subtraction needed in trainings 1 and 2 are captured as well.

Note also that again the mathematical operations and their purposeful application are *integrated.*

Note finally that through connection to counting sequence, “number of” and “change in number of” are now also connected to the important properties of the representation of the count sequence, including *order,* here from smaller to larger.

Children who have not made the mental transition discussed in training 3 may still be able to proceed to some extent by thinking of the addition and subtraction operations factually as in training 1 or through using calculators or computers, as in training 2. But the difficulties already discussed with these training are likely to persist until they make transitions such as just discussed

5) Transition: These steps of transition from verbal to mathematical thinking just discussed would exhibit the type of modifications in engagement discussed earlier as likely when a highly *complex dynamical system* such as the brain develops its capacity for a specific kind of skill, such as at mathematics. The first step (training 3) involves a qualitative change in engagement that in essence begins to develop a *subsystem* devoted to mathematical thinking and its productive application. The next step would then add productive capability involving integer addition and subtraction. These *developmental stages* could be expected to support *jumps* in math performance compared to attempts without the qualitative improvements in engagement. At each of stage engagement operations and capacity for application would become *integrated*. And to the extent that the qualitative changes in engagement increasingly separate developmental path at math by those who make the transitions from those who do not they would involve something like *bifurcations in developmental path***.** But modelling concerning such bifurcations, as Nicolis and Prigogine (1989) emphasize, must be developed very cautiously. Though models of low dimensional systems can and are being used to illustrate and study opportunity for bifurcation in systems with relatively low level of complexity, attempts to extrapolate what these examples show to highly complex systems such as the brain must be developed and tested with great care. It seems likely that bifurcation-like changes in the human brain in particular in its complexity will involve much more complex dynamics of development in its bifurcations than exhibited by simpler systems.

6.3 Training further transformation developing purposeful conceptualization and application of integer multiplication and division

As was the case with addition and subtraction, multiplication and division operations can be taught as involving factual information: 3X 3 = 9, 9/3 = 3. Memorizing multiplication and Division tables was once a typical (and for some unpleasant) step once addition and subtraction had been covered, and indeed memorizing such tables and calculation operations that extent their value can remain useful, though today calculators and computers can in many cases relieve the necessity of carrying out these operations in this way. But as with addition and subtraction using these operations productively may well depend on the degree to which brain operations are developed to lock in their conceptual meaning and value productively. As discussed earlier, most of the teen students I questioned a few years ago could not illustrate the purposeful value of either multiplication or division.

Integer Multiplication and Division can be built from addition and subtraction in several ways. One is to think of these operations as special kinds of applications of addition and subtraction. Multiplication then involves applications where a specified number of equal sized additions are performed, and division a specified number of equal sized subtractions. Whether or not students have already built mental representation relating "change in number of" to operations of addition and subtraction, they can learn to apply memorized information from tables in ways taught in class. And students who have been addressing addition and subtraction in some other way that does not involve the transformations we have been discussing can also try to think of multiplication and division through addition and subtraction as well, though what hampered them earlier is likely to intensify as they try to add these new concepts.

But all students thinking of multiplication and division only in this way can be expected to have difficulty fully understanding the profitable value of these operations conceptually. For at heart multiplication and addition are importantly different conceptually, and the same is true for division and subtraction.

Current Common Core curriculum in math addresses this directly and usefully, presenting multiplication as involving wholes built from equal sized parts, and division as involving wholes that can be broken into equal sized parts.

Thus, importantly, I suggest, from a slightly different perspective, conceptualization of *Multiplication* involves *repetition*, a concept that all children can understand. In building wholes from equal sized components or groups, the size of a component is what is repeated. But integer multiplication as it is applied does not relate as directly to thinking concerning parts and wholes as do changes in "number of" achieved by repeated equally sized increases. Thus integer multiplication can be thought of as involving engagement achieving change of position on the "number of" sequence through repeated movements of equal size rather than by through a series of specific movements each specified by addition. What is now captured conceptually is application involving repetition that a user can immediately understand productively, as in "If I earn $1 every time I mow the lawn, I'll have $20 after I mow it twenty times". This operational treatment of multiplication then readily makes it apparent that only certain integers in the number-of sequence can be reached by integer multiplication.

Integer *Division* as implemented on the "number of" sequence, would then involve attempt to divide the sequence from 1 to a chosen integer into equal sized groups of integers, i.e. to find a way to reach a given total integer by repetitions of equal size moving along the number-of-sequence. This would again capture purposeful application as in "I have eight cups of juice. I can give all eight of you one cup each or if only four want juice, two cups each". And once the limitations of integer multiplication are captured conceptually, the corresponding limitations of integer division would be captured as well. "Sorry, I have nine cups of juice, and so I can't give all four of you the same amount unless you each only get one cup".

By adding these operations of multiplication and division to those of addition and subtraction on the "number of "sequence, conceptualization of operations combining all these operations can be readily developed. And, still more importantly, as the limitations of integer multiplication and division become more apparent, the importance of a further transition to the richer representation of the "number line" (Case, 1985, 1992) and then the need for rational numbers to fill in the gaps becomes apparent. And the student is prepared to extend what has been developed concerning use of repetition in multiplication and division to higher dimensional representation of concern to geometry, to development of uses of fractions, rational numbers, and decimals, and then to algebra.

Building further operations of multiplication and division onto the representation which after training 4 supported purposeful engagement involving addition and subtraction is again, as discussed previously, a transition, a further step to new ways of thinking mathematically in a stage by stage process that as discussed earlier develops operations in a subsystem devoted to building mathematical skill.

The stages discussed here address only a portion of the mathematical skill a child must develop in Elementary schooling, but illustrates and models developments that at every stage integrates mathematical operations with creational capacity for application.

7. Creational Integrated Capability at Skill (CPICS)

Each of the transformations discussed above achieve something similar in support of each stage of mathematical skill development that is examined. They provide in stages qualitative further development of brain activity that produces skill ("engagement", Gardiner, 2008)) in a way that provides for creational application. I refer to what is created at every stage of skill development as *Creational Purposeful Integrated Capability at Skill (CPICS).*

As discussed in the first part of this paper, it appears likely that in a highly complex system such as the human brain, qualitative changes in the engagement producing skillful behavior of any kind will be needed for skill to develop to its greatest potential. The specific changes modelled here may not take place as modelled. But for reasons discussed above, and now further here I think it likely that brain engagement changes that develop what I term CPICS are necessary for skill to reach its highest potential. The specific illustrative modelling is consistent with available evidence, and receives further support to be discussed in a companion paper and previously (Gardiner, 2000, 2008, 2019).

As noted earlier, learning to ride a bicycle illustrates what I mean by CPICS. Other examples can include talking, understanding speech, making sense of the world visually, walking, driving a car, indeed much of the essential skill we have digested sufficiently that we find ourselves performing it without knowing how we are able to do so, or even how we become able to do so. To a greater extent than any other creature, our human skills develop enormously after we are born (Campbell, 1982). What I term CPICS capability builds skill in a way that allows us to adapt it especially profitably to the great complexity of the niche in which we live. It is hard to imagine how we could talk, or do any of the other things for which CPICS capability seems essential if we prepared to execute such skill in a less adaptable way. We are so used to our many CPICS capabilities we do achieve that we can easily take them for granted, just something our remarkable brains are able to do. But I propose in many areas of skill, especially at academic skills that are learned during schooling, CPIS capability is not yet as broadly achieved as may become possible as its basis and means for training it become better understood.

Though the CPICS integration provides what feels like essentially *continuous* real time creational adjustment and development of skillful action during bicycle riding or walking, skillful mental performance even during CPICS engagement is likely to be developed in more *episodic stages.* For example, as addition and subtraction problems come to involve two or more digit integers, or with much of multiplication and division, pencil and paper or electronic calculation will still be needed. What CPICS provides is mental framework that integrates creational application with management of what is needed operationally.

An essential feature of CPICS capability is its integration of *conceptualization* with capacity for application. How we think *consciously* or understand concerning our capability at a skill is a complicated matter that Carey's (2009) evidence and discussion implies is deeply related to how we organize our mental operations concerning that skill. Thus the ability to say whether one number of objects is *greater* than another, and if so by how much, or to even understand the intended meaning of such a question depends on being able to refer to some mental organizational structure such as the number-of sequence to answer the question.

I do not propose that developmental staging such as discussed here is the only way that a brain can build skill at math, or at other skills. But the staging developing CPICS capability at every stage appears to have very useful properties, and this may explain why many capabilities all of us already achieve and honor appear to develop in this way.

The stages of improvement can be thought of as each likely to involve what may be called *bifurcation*, in the sense that capability at skill and its further development is likely to evolve differently and more satisfactorily in those who have made the transition compared to those who have not. As noted previously, Nicolis and Prigogine (1989) warn that the uses of such terminology does not imply that the bifurcations of interest here in the highly complex brain can be modelled or even understood in the same way as those that are being studied in lower dimensional less complex systems. I hope that the examples of modelling presented here, in related research (Gardiner, 2000, 2008, 2019) and in a forthcoming paper can assist in clarifying such changes in brain operations.

8. Conclusion

Attention to the issues discussed should be added to the intense current work seeking to further improve Education at this time when it is so critical to the future of every child. For reasons that are not yet well understood, it seems likely that some children have more difficulty than others in making the mental transitions during schooling discussed here. As illustrated, details of classroom training may help to overcome this. I have proposed as examples possibilities this framework suggests that can continue to be investigated. A companion paper to follow will continue to examine implications, and will also review evidence of striking gains in math capability, especially in weak or at risk students, that interaction with CPICS gains at musical skill as modelled here can help to explain (see also Gardiner, 2019 for a related discussion). These further data also clarify the nature of brain development that CPICS framework addresses.

Education is not only about providing information but also, as discussed here, very possibly about promoting brain changes that foster the development of capabilities of all kinds, in ways that maximize their productive use. The data to be reviewed in the companion paper show evidence that as we improve our understanding of how to promote these changes this can help some and perhaps even many children to advance in their academic skill development more successfully than is presently the case.

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