Northeast Journal of Complex Systems (NEJCS)

Volume 3
Number 1 *Special Issue: NERCCS 2021 Papers*

Article 2

March 2021

Anticipation Induces Polarized Collective Motion in Attraction Based Models

Daniel Strömbom
Lafayette College, stroembp@lafayette.edu

Alice Antia
Carleton College, alice.antia@gmail.com

Recommended Citation

Strömbom, Daniel and Antia, Alice (2021) "Anticipation Induces Polarized Collective Motion in Attraction Based Models," *Northeast Journal of Complex Systems (NEJCS)*: Vol. 3 : No. 1 , Article 2.
DOI: 10.22191/nejcs/vol3/iss1/2
Available at: https://orb.binghamton.edu/nejcs/vol3/iss1/2

Anticipation Induces Polarized Collective Motion in Attraction Based Models

Daniel Strömbom[1,2*] and Alice Antia[3]

[1] Department of Biology, Lafayette College, Easton, PA, USA

[2] Department of Mathematics, Uppsala University, Uppsala, Sweden

[3] Department of Mathematics and Statistics, Carleton College, Northfield, MN, USA

[*] stroembp@lafayette.edu

Abstract

Moving animal groups are prime examples of natural complex systems. In most models of such systems each individual updates its heading based on the current positions and headings of its neighbors. However, recently, a number of models where the heading update instead is based on the future anticipated positions/headings of the neighbors have been published. Collectively these studies have established that including anticipation may have drastically different effects in different models. In particular, anticipation inhibits polarization in alignment-based models and in one alignment-free model, but promotes polarization in another alignment-free model. Indicating that our understanding of how anticipation affects the behavior of alignment-free models is incomplete. Given that attraction is a component of many alignment-free models we include anticipation in an attraction only model here to investigate. We establish that anticipation induces polarized collective motion and inhibits swarming and milling in combination with attraction alone. We also show that anticipation orients milling groups when attraction is sufficiently strong, but not otherwise. Finally, we derive an explicit heading update formula for this model with anticipation that allows for a simple heuristic explanation of its polarization inducing capacity. Due to the biological plausibility of both attraction and anticipation we believe that utilizing these components to explain collective motion in animal groups may be advantageous in some cases.

1 Introduction

Self-propelled particle (SPP) models have been used to model collective motion in a range of animals including fish schools, bird flocks, herds of sheep, and human crowds [1]. SPP models come in a variety of forms and are often characterized by the local interaction rule on which they are based. Common local interaction rules include alignment only [2], attraction only [3,4], attraction and repulsion [5–7], and attraction, repulsion and alignment [8, 9]. While SPP models have been shown to generate a range of different group types the standard groups produced by minimal models are polarized groups, mills and swarms [1, 9]. Polarized groups (also known as dynamic parallel, or aligned, groups) are characterized by collective rectilinear motion resulting from the individuals moving in (approximately) the same direction. Mills are characterized by individuals orbiting a common center, and swarms by individuals moving erratically around a common center. Each of these group types are ubiquitous in nature. For example, fish are known to engage in milling [10], flying insects often engage in swarming [11], and all animal groups that move from one location to another must exhibit at least a degree of polarization or the group would not move.

In most SPP models, regardless of local interaction rule, each particle calculates its new heading based on the current positions and/or headings of its neighbors. However, recently a number of studies have highlighted the potential importance of anticipation in models of this type [12–15]. In models with anticipation, each particle uses the future anticipated positions and/or headings of its neighbors to calculate its new heading rather than current positions and headings. This idea is biologically plausible and it is well established that many animals across taxa use anticipation. For example, predators such as dragonflies [16], bats [17], and hawks [18] use a form of anticipation when pursuing their prey. In addition, humans use anticipation when navigating in crowds [19] and models of pedestrian dynamics that include some type of anticipatory effects are numerous [20–22].

The effects on the resulting collective motion that arises from including anticipation in alignment-based models, i.e. models that include an explicit alignment interaction where particles align their headings with the heading of their neighbors, has been described in [12, 14], and in one attraction and repulsion model without self-propulsion in [13]. The overall finding of these studies is that including anticipation inhibits polarized collective motion and promotes milling and swarming. This is particularly surprising in the case of alignment-based models because the production of polarized collective motion is a key feature of models of this type, whereas production of mills and swarms is not [1]. In addition, [15] has established that polarized collective motion emerges in an alignment-free model based on ”mutual anticipation”, ”following”, ”free movement”, and asynchronous updating on a lattice space. Combined, these examples show that models with different components are differently affected by the inclusion of anticipation, in particular, with respect to their capacity to produce polarized groups.

Attraction is a fundamental biologically plausible interaction in the context of moving animal groups [23] and a key component of many SPP models [1]. In addition, it is a subcomponent of the two alignment-free models introduced above [13,15], of which one produces polarized groups and the other does not. Suggesting that investigating the interplay between attraction and anticipation may help explain the varying effects of anticipation on group formation observed, in particular, its polarization inducing capacity. Here we introduce anticipation of the type used in [13] in a version of the local attraction model [4]. This model is particularly suitable for investigating the effects of anticipation in combination with attraction because particles interact via attraction alone and it is the simplest model known to produce the three standard groups, non-oriented mills, swarms and polarized groups, under certain conditions [3]. In particular, it generates all three groups when asynchronous update is used and only non-oriented mills and swarms when synchronous update is used [4]. Here we introduce anticipation into the synchronous version because it is more restrictive with respect to group formation, in particular, it does not produce polarized groups, and most SPP models against which it should be compared have been implemented with synchronous update only [4].

2 Model and Methods

First we provide a summary of the local attraction model (LAM) [3, 4], including a previously unpublished derivation of the heading update term in this model, and then we describe how anticipation was introduced. We use hat notation for normalized vectors and bar notation for vectors of arbitrary length.

The LAM is a SPP model in which particles interact via local attraction alone (Fig 1A). We denote the position of particle i at time t by P_i^t, the set of indices of particle i's neighbors at time t by Ω_i^t, where i's neighbors are all other particles within a distance of R from it, and the number of neighbors at time t by $|\Omega_i^t|$. The local center of mass that particle i detects at time t is given by

$$LCM_i^t = \frac{1}{|\Omega_i^t|} \sum_{j \in \Omega_i^t} P_j^t \tag{1}$$

and the non-normalized local attraction vector from P_i^t to the LCM_i^t is

$$\bar{C}_i^t = LCM_i^t - P_i^t = \frac{1}{|\Omega_i^t|} \sum_{j \in \Omega_i^t} P_j^t - P_i^t \tag{2}$$

and the non-normalized current heading vector of particle i is

$$\bar{D}_i^t = P_i^t - P_i^{t-1}. \tag{3}$$

The heading update formula for particle i is

$$\bar{D}_i^{t+1} = c\widehat{C}_i^t + \widehat{D}_i^t \tag{4}$$

where the parameter c specifies the relative tendency to steer towards the LCM_i^t when the relative tendency to proceed with the current heading is 1, and the positional update formula is

$$P_i^{t+1} = P_i^t + \delta \frac{c\widehat{C}_i^t + \widehat{D}_i^t}{|c\widehat{C}_i^t + \widehat{D}_i^t|} \tag{5}$$

where δ is the displacement of the particle per time step. This is the model studied in [4] with asynchronous update (particles update their headings and positions in sequential random order) and synchronous update (all particles update their headings and positions simultaneously).

We now extend the LAM to include anticipation and use a preindex a to distinguish components of the model with anticipation from the original model without anticipation. For example, the position of particle i in the original model is denoted by P_i^t and the anticipated position of particle i in the model with anticipation is denoted by ${}_aP_i^t$. Following [13] we define the anticipated position of a neighboring particle j by

$${}_aP_j^t = P_j^t + \tau\delta\widehat{D}_j^t \tag{6}$$

where τ is the anticipation time. The anticipated position of a particle is therefore the position that that particle would be at if it continued with its current heading for τ time steps, and particle j is a neighbor of particle i if ${}_aP_j^t$ is within a distance of R from P_i^t. See Fig 1B. The formula for the anticipated local center of mass detected by particle i at time t is given by

$${}_aLCM_i^t = \frac{1}{|{}_a\Omega_i^t|} \sum_{j \in {}_a\Omega_i^t} {}_aP_j^t = \frac{1}{|{}_a\Omega_i^t|} \sum_{j \in {}_a\Omega_i^t} (P_j^t + \tau\delta\widehat{D}_j^t) \tag{7}$$

where $|{}_a\Omega_i^t|$ is the number of anticipated neighbors. The non-normalized local interaction vector with anticipation is

$$\overline{{}_aC}_i^t = {}_aLCM_i^t - P_i^t = \frac{1}{|{}_a\Omega_i^t|} \sum_{j \in {}_a\Omega_i^t} P_j^t - P_i^t + \tau\delta \frac{1}{|{}_a\Omega_i^t|} \sum_{j \in {}_a\Omega_i^t} \widehat{D}_j^t. \tag{8}$$

The heading update formula for particle i in the LAM with anticipation is then given by

$$\overline{{}_aD}_i^{t+1} = c\widehat{{}_aC}_i^t + \widehat{D}_i^t. \tag{9}$$

We note that this reduces to the heading update formula for the LAM without anticipation (Eq. 4) when $\tau = 0$ because then $\widehat{{}_aC}_i^t = \widehat{C}_i^t$. Finally the positional update formula for the LAM with anticipation is

$$P_i^{t+1} = P_i^t + \delta \frac{c_a \widehat{C}_i^t + \widehat{D}_i^t}{|c_a \widehat{C}_i^t + \widehat{D}_i^t|}. \tag{10}$$

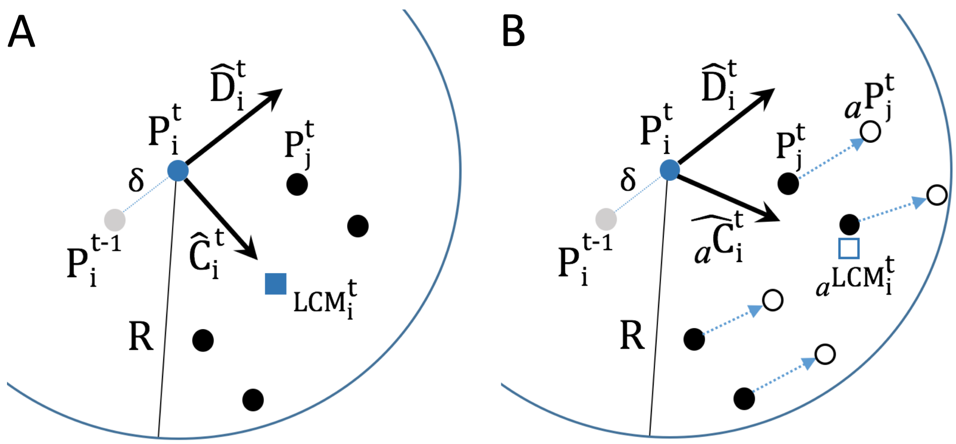

Figure 1: The LAM with and without anticipation. Illustration of how the local attraction vector ($\widehat{C}_i^t$) and current heading vector ($\widehat{D}_i^t$) are set up for a focal particle i located at position P_i^t in the LAM without anticipation (A) and with anticipation (B). The main difference between the two versions is that the local center of mass (LCM) is the actual center of mass of the neighbors in the model without anticipation (LCM_i^t), and an anticipated future center of mass in the model with anticipation (${}_aLCM_i^t$). This affects the calculation of the local attraction vector but leaves the current heading vector unchanged. These figures have been adapted from Fig 2a in [4] (Strömbom CC-BY).

2.1 Simulations and Analysis

We analyzed the model via simulation and used two standard measures to quantify the polarization and size of the resulting groups.

The polarization, or velocity alignment, (α) measures the degree to which all the particles are heading in the same direction and can be calculated via the formula

$$\alpha = \frac{1}{N} \left| \sum_{i=1}^{N} \widehat{D}_i \right|, \tag{11}$$

where N is the total number of particles and $\widehat{D}_i$ is the normalized current heading of particle i [2]. The polarization ranges from 0 to 1 and the higher the value the more polarized the group is. In particular, highly polarized groups have polarization value 1, and mills, where the particles orbit a common center, have low polarization values (close to 0) because the sum of current heading vectors ($\widehat{D}_i$) cancel out when the particles are moving in a circular path.

The scaled size (σ) measures how much of the available space the group occupies and can be calculated by the formula

$$\sigma = \frac{(\Delta P_x)(\Delta P_y)}{L^2}, \tag{12}$$

where ΔP_x is the length of the range of particle x-coordinates, ΔP_y is the length of the range of particle y-coordinates, and L^2 is the total area of the simulation environment [3, 4]. Scaled size ranges from 0 to 1 and the more space the group occupies the higher the value. In particular, if no group has formed σ is high (≈ 1) and if a cohesive group has formed σ is lower.

Using these measures we can determine if a cohesive group has formed and whether or not it is a polarized group. Allowing us to distinguish between the known group types: no group, cohesive polarized group, and the mill and swarm phases. See [4] for further details.

To analyze the model we ran 100 simulations for each (c, τ)-pair with c and τ varying from 0 to 2 in increments of 0.1. To facilitate comparison with [4] we use $N = 50$, $R = 4$ and $\delta = 0.5$. The particles move in a 2D region with periodic boundary conditions and at the start of each simulation the N particles are assigned random positions and headings. Each simulation was 10000 time steps long and over the last 50 time steps of each simulation the polarization (α) and scaled size (σ) of the resulting group was measured, and the mean of the 50 values for both measures were returned from the simulation.

We also generated plots showing the distribution of polarization values returned for each value of c over a 100 simulations for $\tau = 0$, $\tau = 1$ and $\tau = 2$. To achieve this we partitioned the range of the polarization measure $[0, 1]$ into 50 subintervals of equal length and counted the number of polarization values returned from simulations in each subinterval and scaled the result for each c by the total number of simulations (100). This type of plot will show if two polarization-wise distinct groups tend to form for a given c, for example, one with high polarization and one with low polarization (depending on the initial conditions). Information that would not be apparent if the average polarization for each c was displayed instead.

3 Results

The model with anticipation ($\tau = 1$) generates polarized groups, non-oriented mills, and oriented mills. Unlike the original model without anticipation ($\tau = 0$) that only produces no group, non-oriented mills and swarms [4]. In Fig 2 examples of the groups that form for different values of c are displayed and their polarization and scaled size values specified. For $c = 0.1$ the model with anticipation produces polarized groups and the original model produces no group. For $c = 0.5$ and $c = 1$ both models produce non-oriented mills that appear similar, however, after 10000 time steps there is a large difference in the polarization values. The mills produced by the original model have polarization values on the order of 10^{-14} whereas the mills produced by the model with anticipation is on the order of 10^{-2}. For $c = 1.5$ the original model produces non-oriented mills and the model with anticipation generates non-oriented mills in some simulations and polarized groups in others. Finally, for $c = 2$ the model with anticipation generates oriented mills in some simulations and polarized groups in others, whereas the original model only generates swarms.

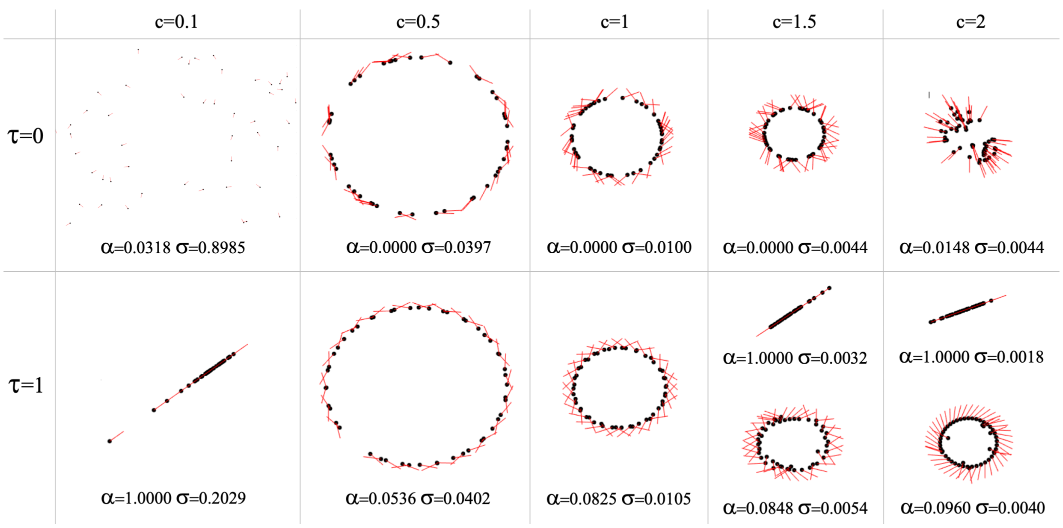

Figure 2: Group types produced with anticipation and without anticipation. In the model without anticipation ($\tau = 0$) no group is produced when $c = 0.5$, non-oriented mills when $c = 0.5{-}1.5$, and swarms when $c = 2$. This is consistent with the results of the synchronous update model in [4]. In the model with anticipation ($\tau = 1$) polarized groups form when $c = 0.5$, non-oriented mills when $c = 0.5 - 1$, a mix of non-oriented mills and polarized groups when $c = 1.5$, and a mix of oriented mills and polarized groups when $c = 2$. The black dots represent the particles and the red rods the $\widehat{D}_i^t$ vector in Fig 1.

The distribution of polarization values returned for each c from 0 to 2 in increments of 0.1 for $\tau = 0$, $\tau = 1$ and $\tau = 2$ is shown in Fig 3, and the result is consistent with the picture presented in Fig 2. In particular, it confirms that for c large both mills and polarized groups form in the model with anticipation for $\tau = 1$. In this case there is substantial production of both mills (α low) and polarized groups (α high), but no sign of groups with intermediate polarization. Indicating that either a mill or a polarized group form in each simulation and that the outcome depends on the initial conditions. Comparing the polarization distributions for $\tau = 0$, $\tau = 1$ and $\tau = 2$ in Fig 3 also reveal that the proportion of polarized groups relative to mills, for c large, increases with τ, and for $\tau = 2$ mills are no longer produced, only polarized groups emerge for $c \geq 1.4$.

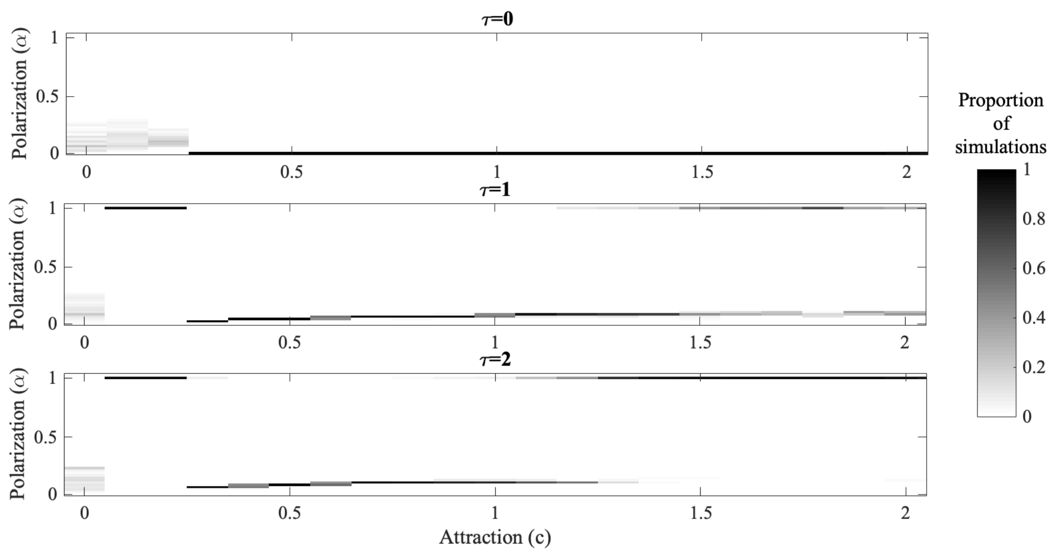

Figure 3: Polarization distributions over c for $\tau = 0$, 1 and 2. When there is no anticipation ($\tau = 0$) polarization remains low for all values of c indicating that polarized groups do not form. As τ is increased to 1 polarized groups start to emerge for $c \leq 0.2$ and $c \geq 1.3$, and in the latter range both polarized and non-polarized groups are readily produced for each value of c. When τ is increased to 2 the polarization for $c \leq 0.2$ is unaffected, but for $c \geq 1.4$ a mix of groups is no longer produced, instead the model exclusive produce polarized groups for these values of c.

A more detailed investigation of the group formation over the (c, τ) parameter space with τ varying from 0 to 2 in increments of 0.1 shows that there are indeed two distinct regions where cohesive polarized groups are consistently produced. One for $c \leq 0.2$ and one for larger c. See Fig 4. In between these regions cohesive non-polarized groups are consistently generated. It also shows that a significant anticipation time τ is required to generate polarized groups both for small and large c, and that this time is higher for polarization to emerge in the large c region.

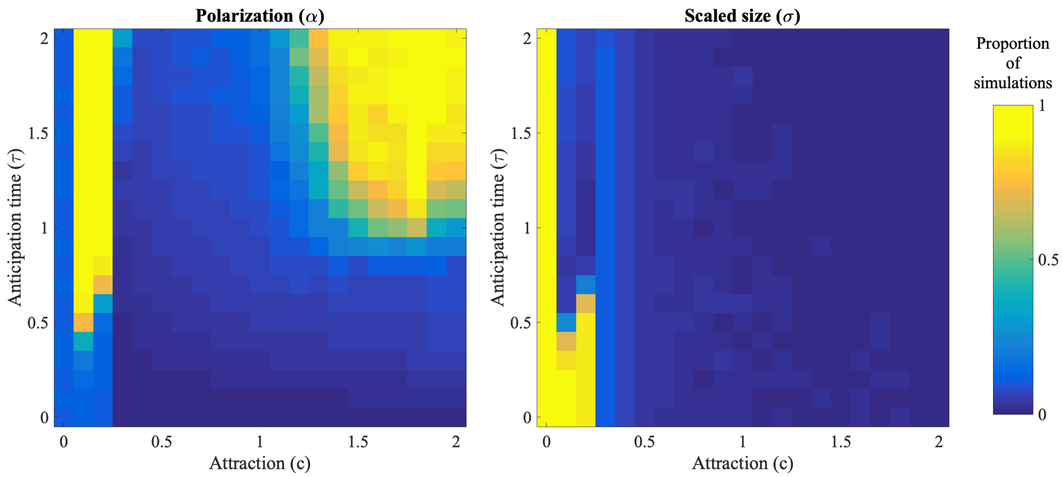

Figure 4: Group formation over the (c, τ) parameter space. The polarization (α) shows that there are two regions of high polarization, one for small c and one for larger c, separated by a region of low polarization. The scaled size (σ) shows that there is one region where the scaled size is high, indicating that no group formed for those parameter combinations, and for all other combinations cohesive groups tend to form. Combined this shows that cohesive polarized groups form in two distinct regions in the included parameter ranges.

How the inclusion of anticipation ($\tau > 0$) induces a polarizing effect can be heuristically understood via the mathematical description of the model (Eq. 6-10). The interaction vector $\overline{{}_aC}_i^t$ (Eq 8) contains all the interactions between particles in the model with anticipation. It consists of one attraction part ($\frac{1}{|{}_a\Omega_i^t|}\sum_{j\in {}_a\Omega_i^t} P_j^t - P_i^t$) that we denote by $\overline{K}_i^t$, and one anticipated alignment part ($\tau\delta\frac{1}{|{}_a\Omega_i^t|}\sum_{j\in {}_a\Omega_i^t} \widehat{D}_j^t$) that we denote by $\overline{A}_i^t$. Intuitively, one expects that if $|\overline{K}_i^t| > |\overline{A}_i^t|$ the interactions are dominated by attraction and the model effectively reduces to the model without anticipation. Conversely, if $|\overline{A}_i^t| > |\overline{K}_i^t|$ one expect that anticipated alignment will dominate and particles will align their headings with the average anticipated heading of the anticipated neighbors and polarization might emerge. We also note that the magnitude of the anticipated alignment term is $\tau\delta$ which increases with τ for δ fixed. This, at least heuristically, explains why the polarization inducing capacity of the model increases with τ and why $\tau = 0$ yields no polarization in this version of the model.

4 Discussion

How moving animal groups coordinate themselves via local interactions between individuals is typically explained via a combination of attraction, repulsion and alignment using the current positions and headings of neighboring particles to define these interactions. Recently, using arguments pertaining both to biological plausibility and model theoretical considerations a number of authors have argued that it might be advantageous to use anticipated future positions and headings rather than the current to explain a number of phenomena observed in moving animal groups. In particular, the emergence of polarized collective motion. However, the outcome of adding anticipation to a number of different models have been disperate, suggesting that our understanding of the effects of anticipation, in particular, in alignment-free models, is lacking. Here we included positional anticipation into an attraction only model, because attraction is a fundamental biological interaction and a core component of many models of collective motion, to investigate.

We established that introducing anticipation in combination with attraction only induces polarized group formation and inhibits swarm and mill formation. Therefore, including anticipation in combination with attraction has the opposite effect that including anticipation in combination with alignment has been reported to have. Namely that polarization is inhibited and milling and swarming are promoted [12–14]. Our work also provide a potential explanation for the polarization inducing capacity of the model in [15] that contains both attraction-like interactions and anticipation. However, this model employ asynchronous updating, in addition to anticipation, that is also known to induce polarization in combination with attraction alone [4]. Therefore, it remains unclear exactly which components of the model in [15] is responsible for the polarization inducing capacity of the model.

The finding that the magnitude of the anticipated alignment is given by $\delta\tau$ is interesting to consider in relation to the findings in [13]. The original model is an attraction and repulsion model with self-propulsion that readily produce polarized groups [5], however, when the self-propulsion is removed and anticipation is included polarization is inhibited in favor of milling and swarming [13]. Based on this observation and our work here suggest that anticipation, self-propulsion, and speed are intimately linked in alignment-free models and that exploring this link further may advance our understanding of the inclusion of anticipation in alignment-free models more broadly.

In [4] it was established that asynchronous update induced polarized group formation where no group would form with synchronous updates, but it left the milling and swarm regimes largely unaffected. Here we have shown that including anticipation not only induce polarization where no group forms, but also where mills and swarms form, in the original model. Indicating that anticipation is a stronger polarization inducing mechanism than asynchrony. Hence, at least in models similar to ours where collective motion is driven by attraction, anticipation may be an underutilized mechanism for inducing polarized collective motion. This observation may be particularly useful to consider in the context of inferring interaction rules in real animal groups from trajectory data. In particular, where some studies find no evidence for standard alignment interactions operating in schools of fish [24, 25] while others do [26]. While there could be many other circumstances that better explain this, as discussed in [4, 7], adding anticipation to the list of mechanisms that can generate polarized collective motion in SPP models, in particular as an alternative to explicit alignment interactions, may be useful in advancing our understanding of how qualitatively similar collective motion emerges from seemingly different local interactions between individuals in the group.

Acknowledgements

This work was in part supported by a grant from the Swedish Research Council to D.S. (ref: 2015-06335).

References

[1] Vicsek T, Zafeiris A. Collective motion. Physics Reports. 2012;517:71–140.

[2] Vicsek T, Czirók A, Ben-Jacob E, Cohen I, Shochet O. Novel type of phase transition in a system of self-driven particles. Physical review letters. 1995;75(6):1226.

[3] Strömbom D. Collective motion from local attraction. J Theor Biol. 2011;283:145–151.

[4] Strömbom D, Hassan T, Hunter Greis W, Antia A. Asynchrony induces polarization in attraction-based models of collective motion. Royal Society Open Science. 2019;6(4):190381.

[5] D'Orsogna MR, Chuang YL, Bertozzi AL, Chayes LS. Self-propelled particles with soft-core interactions: patterns, stability, and collapse. Physical review letters. 2006;96(10):104302.

[6] Romanczuk P, Schimansky-Geier L. Swarming and pattern formation due to selective attraction and repulsion. Interface focus. 2012;2(6):746–756.

[7] Strömbom D, Siljestam M, Park J, Sumpter DJ. The shape and dynamics of local attraction. The European Physical Journal Special Topics. 2015;224(17-18):3311–3323.

[8] Reynolds CW. Flocks, herds and schools: A distributed behavioral model. SIGGRAPH Comput Graph. 1987;21:25–34.

[9] Couzin ID, Krause J, James R, Ruxton GD, Franks NR. Collective memory and spatial sorting in animal groups. J Theor Biol. 2002;218:1–11.

[10] Tunstrøm K, Katz Y, Ioannou CC, Huepe C, Lutz MJ, Couzin ID. Collective states, multistability and transitional behavior in schooling fish. PLoS Comput Biol. 2013;9(2):e1002915.

[11] Attanasi A, Cavagna A, Del Castello L, Giardina I, Melillo S, Parisi L, et al. Collective behaviour without collective order in wild swarms of midges. PLoS Comput Biol. 2014;10(7):e1003697.

[12] Morin A, Caussin JB, Eloy C, Bartolo D. Collective motion with anticipation: Flocking, spinning, and swarming. Physical Review E. 2015;91(1):012134.

[13] Gerlee P, Tunstrøm K, Lundh T, Wennberg B. Impact of anticipation in dynamical systems. Physical Review E. 2017;96(6):062413.

[14] Baggaley AW. Stability of model flocks in a vortical flow. Physical Review E. 2016;93(6):063109.

[15] Murakami H, Niizato T, Gunji YP. Emergence of a coherent and cohesive swarm based on mutual anticipation. Scientific Reports. 2017;7.

[16] Olberg RM. Visual control of prey-capture flight in dragonflies. Current opinion in neurobiology. 2012;22(2):267–271.

[17] Ghose K, Horiuchi TK, Krishnaprasad P, Moss CF. Echolocating bats use a nearly time-optimal strategy to intercept prey. PLoS biology. 2006;4(5):e108.

[18] Kane SA, Fulton AH, Rosenthal LJ. When hawks attack: animal-borne video studies of goshawk pursuit and prey-evasion strategies. Journal of Experimental Biology. 2015;218(2):212–222.

[19] Karamouzas I, Skinner B, Guy SJ. Universal power law governing pedestrian interactions. Physical review letters. 2014;113(23):238701.

[20] Johansson A. Constant-net-time headway as a key mechanism behind pedestrian flow dynamics. Physical review E. 2009;80(2):026120.

[21] Moussaïd M, Helbing D, Theraulaz G. How simple rules determine pedestrian behavior and crowd disasters. Proceedings of the National Academy of Sciences. 2011;108(17):6884–6888.

[22] Lü YX, Wu ZX, Guan JY. Pedestrian dynamics with mechanisms of anticipation and attraction. Physical Review Research. 2020;2(4):043250.

[23] Ward A, Webster M. Sociality: the behaviour of group-living animals. Springer; 2016.

[24] Herbert-Read JE, Perna A, Mann RP, Schaerf TM, Sumpter DJ, Ward AJ. Inferring the rules of interaction of shoaling fish. Proceedings of the National Academy of Sciences. 2011;108(46):18726–18731.

[25] Katz Y, Tunstrøm K, Ioannou CC, Huepe C, Couzin ID. Inferring the structure and dynamics of interactions in schooling fish. Proceedings of the National Academy of Sciences. 2011;108(46):18720–18725.

[26] Gautrais J, Ginelli F, Fournier R, Blanco S, Soria M, Chaté H, et al. Deciphering interactions in moving animal groups. PLoS Comput Biol. 2012;8(9):e1002678.