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1 Abstract

This work presents an innovative approach to understanding and measuring complexity in network models. We revisit several classic characterizations of complexity and propose a novel measure that represents complexity as an interactive process. This measure incorporates transfer entropy and Jensen-Shannon divergence to quantify both the information transfer within a system and the dynamism of its constituents' state changes.

To validate our measure, we apply it to several well-known simulation models implemented in Python, including: two models of residential segregation, Conway's Game of Life, and the Susceptible-Infected-Susceptible (SIS) model. Our results reveal varied trajectories of complexity, demonstrating the efficacy and sensitivity of our measure in capturing the nuanced interplay of interactivity and dynamism in different systems. The results corroborate the notion that heterogeneity and stochasticity increase system complexity.

This study contributes to the field by proposing a measure that not only quantifies the amount of complexity present in a system but also emphasizes the process of ‘‘complexing,‘‘ marking a semantic shift from viewing complexity solely as an attribute or condition. Our findings underscore the significance of considering both interactivity and dynamism in defining and measuring complexity. The study also acknowledges limitations related to computational resources and the simplification of transfer entropy calculations, setting a clear path for future research in refining and expanding this measure of complexity.

2 Introduction

As our world becomes increasingly interconnected with uncountable components and interactions, the importance of complexity science increases commensurately.

Naturally, ideas, definitions, and measures of complexity have concurrently grown in abundance in the literature. In his classic illustration of complexity, Weaver [1] identified three regimes in which problems reside: organized simplicity, where a small number of possibly interacting components are observed; organized complexity, where larger number of interacting components are observed; and disorganized complexity where a perhaps massively large number of components are interacting with some stochasticity are observed. Moving beyond these categorical characterizations of complexity, we seek measures by which to quantify it. Constructing such a description requires a definition or concept of complexity.

Mitchell [2], acknowledging the challenge inherent in selecting a single definition or conceptualization, reviews several characterizations of complexity: size, entropy, algorithmic information content, logical depth, thermodynamic depth, computational capacity, statistical complexity, fractal dimension, and degree of hierarchy. Excepting algorithmic information content, each are system attributes which can be directly measured from its static image. Gershenson [3] suggests that complexity can be found when a system's state dynamics are balanced between self-organization and emergence in the observed variety of states that a system or its components may take over a time period. De Dominico et al. [4] assert that complexity is evident where interactivity, emergence, dynamics, self-organization, and adaptation are observed. These concepts emphasize interactivity and dynamism whose measures require observation over time. Thompson et al. [5] emphasize the importance of interactivity in complexity in their work to develop a measure of complexity in simulation models.

The descriptions we, as observers, may construct are dependent upon the language available to us. The difficulty we experience in distilling complexity as a concept may be a result of a tendency in the literature to articulate complexity as an attribute or condition, rather than as a process. Such semantic choices have been shown to influence the scientific process [6]. The verb form of the term "complex" typically refers to the formation of complexes in chemistry and is not frequently encountered in the context of complexity science. This tendency may bias our system description toward complexity as an attribute and away from "complexing" as a process. Each of the sources previously discussed which sought to refine complexity as a concept noted the importance of interactivity as a necessary condition for complexity [2, 4, 5]. However, none proposed concrete measures which required evidence of interactivity. Algorithmic complexity may represent interactivity, however, identifying the shortest algorithmic description of a system is not a trivial task. The other measures emphasized how much complexity a system "has" rather than how much complexing it is "doing." We propose to aid a semantic shift by providing a measure of complexity which emphasizes meaningful interactivity. The measure proposed in the next section accounts for these shortcomings.

3 Proposing a Measure

The objective of this work is not to proclaim a superior measure of complexity, but rather to propose a measure which complements existing measures, thereby enabling a fuller characterization of complexity in systems. To do so, our measure is designed to elucidate meaningful interactivity.

In constructing our measure, it is important to note that, per [7], a system is dependent upon the observer and the system description they choose to adopt. Returning to Weaver [1], at first glance, it appears that the case of disorganized complexity would necessarily be substantially more complex than the case of organized complexity. However, we needn't reduce the case of disorganized complexity to its constituent components and engage in a transcomputational task. We can instead alter our description of the system and, viewing it at a higher level, employ methods from statistical mechanics. We see immediately that the complexity of a system depends on the description constructed by its observer. Thus, in measuring a system's complexity, our measure must be able to reflect this observational selectivity. This may include the specification of components, the choice of sampling frequency, or the scale(s) at which the system is observed.

3.1 Interactivity

To measure interactivity within a system, we employ transfer entropy as described by Schreiber [8]:

$$T_{X \to Y} = \sum p(Y_{t+1}, y_t^k, x_t^l) \frac{p(y_{t+1}|y_t^k, x_t^l)}{p(y_{t+1}|y_t^k)} \tag{1}$$

where y_{t+1} is the future state Y, y_t^k is the past k states of Y up to time t, x_t^l is the past l states of X up to time t, $p(Y_{t+1}, y_t^k, x_t^l)$ is the joint probability of the coincidence of these states, $p(y_{t+1}|y_t^k, x_t^l)$ is the conditional probability of the future state of Y given the past states of both X and Y, and $p(y_{t+1}|y_t^k)$ is the conditional probability of the future state of Y given only its own past states. Effectively, this measure indicates the improvement made upon the prediction of the future states of Y given the past states of X, beyond what is already predicted by the past states of Y. Thus, it is an indication of the transfer of information from X to Y. This measure satisfies Klir's [9] requirement that an informational measure of a system's complexity be proportional to the amount of information required to describe the system since $T_{X \to Y}$ is upper-bounded by the size of Y's state-space. Since transfer entropy is computed from a specific time series, it is also sensitive to the sampling frequency of the time series.

Averaging over a specified window of time, we obtain:

$$T_{X\to Y}(\Delta t) = \frac{1}{\Delta t}\sum_{t=0}^{\Delta t-1}\sum p(y_{t+1}, y_t^k, x_t^l)\frac{p(y_{t+1}|y_t^k, x_t^l)}{p(y_{t+1}|y_t^k)} \tag{2}$$

For a system, S, we can find the pairwise transfer entropy:

$$T_{pairs}(S, \Delta t) = \sum_{i\in S}\sum_{j\in S, j\neq i} T_{i\to j}(\Delta t) \tag{3}$$

Certain limitations impair our use of pairwise transfer entropy. Pairwise transfer entropy may overestimate information transfer unless conditioned on background processes. This leads to a situation with infeasible computational demands. This problem worsens when all possible hyperedges are considered.

To avoid this obstacle, we elect to pursue a measure which examines the information between individuals and their respective neighborhoods. In doing so, we view the collection of neighbor states as a whole. A dynamic mapping is used to transform each unique collection to a corresponding unique integer value. Thus, each individual, i, has a neighborhood, n_i, the set of nodes connected to i by at least one edge. Then,

$$T_{neighborhood}(S, \Delta t) = \sum_{i\in S} T_{n_i\to i}(\Delta t) \tag{4}$$

This method appears to be a safer alternative for use with the models we have selected since any information relayed from individuals outside the neighborhood must be transmitted through a member of the neighborhood. Further work is required to develop a comprehensive methodology applicable to a broader set of models. This methodology has the additional advantage that it can be easily adapted to networks with dynamism in their topologies. To further reduce the complexity of our computations, we selected $k = l = 1$ for our calculations.

3.2 Meaning (Dynamics)

Interactivity alone is insufficient. Per [4], a system must exhibit dynamism in order to be considered complex. Imagine two systems, each with four possible states and for some time period, Δt, we have $T_{pairs}(S_1, \Delta t) = T_{pairs}(S_2, \Delta t)$ with distributions: $P_{S_1} = P_{S_2} = [.25, .25, .25, .25]$. It is possible to conceive of a scenario in which at the next time step, $P_{S_1} = [.24, .25, .25, .26]$ and $P_{S_2} = [.1, .4, .1, .4]$. Here we find that interactivity is equivalent, but this activity has resulted in very little real change to the elements of S_1 while substantial change has manifest to the elements of S_2. A natural next step would be to examine the change in Shannon entropy, $H = -\sum_{i\in P_S} p_i \log p_i$, from t to $t + 1$ [10]. Unfortunately, a pitfall arises: imagine that at time $t+2$, $P_{S_2} = [.4, .1, .1, .4]$. Then, $H(P_{S_2}(t+1)) = H(P_{S_2}(t+2))$, despite a real, substantial change in the system's elements. To ameliorate this, one might instead employ Kullback-Leibler divergence:

$$D_{KL}(P||Q) = \sum_{x \in X} P(x) \log \frac{P(x)}{Q(x)} \tag{5}$$

where P is future distribution of the system and Q is the prior distribution of the system [11]. This provides a measure of distance between the future and prior distributions. Unlike Shannon entropy, $D_{KL}(P_{S_2}(t+1)||P_{S_2}(t+2)) > 0$. Unfortunately, another pitfall arises when at $t+3$, $P_{S_2} = [.4, 0, .1, .4]$ because D_{KL} is undefined when a state disappears from or when a novel state appears in a distribution.

To contend with this issue, we instead employ Jensen-Shannon divergence:

$$D_{JS}(P||Q) = \frac{1}{2} D_{KL}(P||M) + \frac{1}{2} D_{KL}(Q||M) \tag{6}$$

where M is the mean distribution calculated from P and Q [12]. This measure is directionless and bounded between 0 and 1 where a value of 0 represents no difference in the distributions and a value of 1 represents maximal difference. Thus, D_{JS} provides a reliable measure of dynamism amongst a system's constituent components independent of system size.

It is important to note that D_{JS} alone is similarly insufficient. Without the presence of interactivity, the observed change in the system cannot be related to the interactivity amongst its constituents, but rather must be the result of some other cause. In this case, the chosen system description should be replaced with one more reflective of the complexity observed. More concretely, we suggest that a good system description requires that its complexing and complexity be aligned.

3.3 Interactive Complexity

Here, we propose a new measure of complexity as the product of the pairwise transfer entropy and the corresponding Jensen-Shannon divergence:

$$C = T_{neighborhood}(S, \Delta t) \times \sum_{t}^{\Delta t} D_{JS}(P_S(t+1)||P_S(t)) \tag{7}$$

This product is minimized when either interactivity is not evident or when dynamism in the components' states is not evident. Conversely, this product is maximized when both interactivity and dynamism are prominent in the system's elements.

This formulation ensures that we do not erroneously identify complexity where dynamism is driven by externalities. Imagine a set of traffic lights changing on a schedule set by a novice. These lights may coincidentally exhibit dynamism in their states, but that dynamism will not be a result of their interaction. It also ensures that we only make claims of complexity when interactivity manifests dynamism. Imagine now that the traffic lights self-organize, as in [13], but vehicles are continually added to the system until it is perfectly congested. As the system is filled, interactivity increases until a failure point is reached and the interactivity can no longer manifest changes in the system. After this threshold, there is no longer any measurable information transfer. This raises an important point: dynamism is a necessary condition for interactivity in this formulation as $D_{JS} = 0 \implies T_{neighborhood} = 0$.

4 Simulations

To further illustrate this proposed measure of complexity, several well-known simulation models were implemented and observed. For all cases, the NetworkX package in Python was used to instantiate a 20x20 regular lattice [14]. Individual cases varied by rule sets and, in some cases, neighborhood size and/or boundary conditions. For each model, a batch of 25 distinct simulations over 500 time steps was generated and executed. In each case, complexity was measured on the distribution of neighborhood compositions. To measure transfer entropy, we used the time-local variant of transfer entropy ($\Delta t = 1$) from the Pyinform package [15]. To compute entropy and divergence, we used the SciPy package [16]. Thus, our computation of interactive complexity, as the product of these, relied on these packages as well.

4.1 Segregation

Two network models of residential segregation were examined: a simple, fast-converging implementation proposed by [17] and popularized by [18], and a more sophisticated model proposed by [19].

4.1.1 Schelling Model

The Schelling [18] model is implemented on a regular lattice with von Neumann neighborhoods and closed boundary condition. Nodes are either populated by a red or blue individual, or left vacant. For our implementation, the vacancy rate was 0.15. Individuals were assigned uniform tolerances, ϵ, for mixing in their respective neighborhoods. For more direct comparison with the Xie & Zhou model [19], we set ϵ at approximately 0.4, meaning that individuals with more than roughly 40 percent dissimilar neighbors, the individual will be unhappy. At each time step, an unhappy individual is randomly select and transferred to an eligible vacant node where its tolerance threshold is not exceeded. This process repeats until either all nodes are satisfied with this position or no favorable trades remain.

4.1.2 Schelling Results

As shown in Figure 1, transfer entropy, Jensen-Shannon divergence, and complexity are prominent early before collapsing at near the 200^{th} time step. This is consistent with our understanding of this particular model. We can expect to see strong interactivity with initial random mixing that subsides once individuals are largely situated in their desired neighborhoods. Modest acceleration in complexity is propelled by strengthening of neighbor influences as organization increases. We observe low levels of Jensen-Shannon divergence at each time step which is consistent with the asynchronous update procedure used in this implementation - each update can, at most, impact 1% of neighborhoods. In this deterministic example, the only dynamical motivator is interactivity, and we see both of these phenomena collapse concurrently.

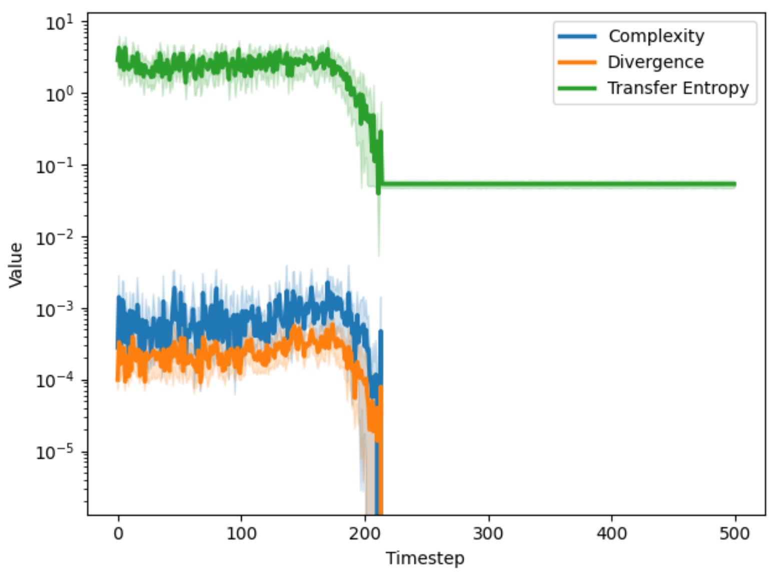

Figure 1: Jensen-Shannon Divergence, Transfer Entropy, and Complexity results for Schelling model simulations.

4.1.3 Xie & Zhou Model

The Xie & Zhou [19] model expands upon its predecessor by representing heterogeneous preferences amongst individuals. For 10.47% of individuals, ϵ fell within

[0.0,0.07); for 18.10% of individuals, ϵ fell within [0.07,0.21); for 26.73% of individuals, ϵ fell within [0.21, 0.36); for 13.86% of individuals ϵ fell within [0.36,0.57); for 26.59% of individuals, ϵ fell within [0.057,1.00]. For the remaining 4.25% of individuals, a rank-ordered logit model was constructed to account for individuals found not to conform to the Guttman scale. See [19] for additional details related to their implementation. The introduction of heterogeneity in this implementation extends complexity. Unlike the Schelling model, this simulation is not halted at equilibrium due to the presence of individuals with stochastic payoff functions.

4.1.4 Xie & Zhou Results

The results of the Xie & Zhou simulations in Figure 2 are distinct from those obtained via Schelling simulations. These results show steadier and more protracted Jensen-Shannon divergence, transfer entropy, and complexity. This aligns with our understanding of this model. The introduction of both stochasticity and heterogeneity enables protracted complexity throughout the simulation. The levels of transfer entropy, divergence, and complexity are similar to those in the Schelling simulation runs before the 200^{th} time step. This again illustrates the complexity boundary associated with the asynchronous update rule for this model.

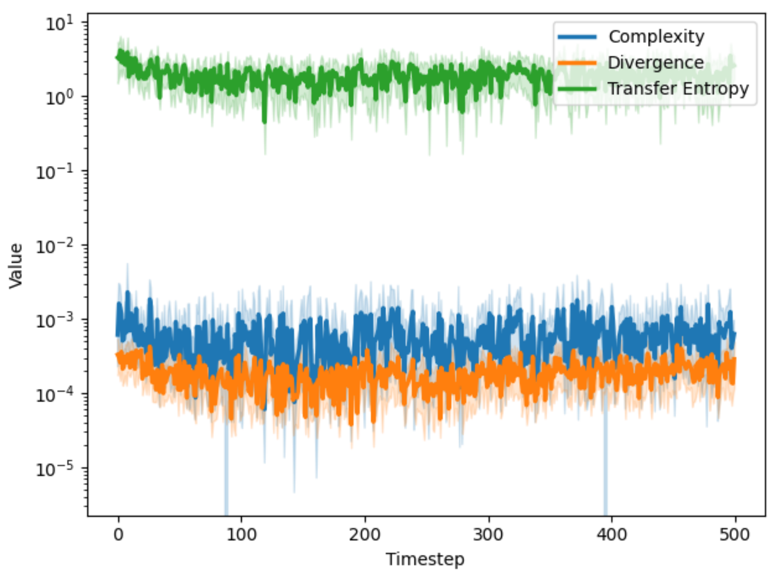

Figure 2: Jensen-Shannon Divergence, Transfer Entropy, and Complexity results for Xie & Zhou model simulations.

4.2 Game of Life

4.2.1 Model

Conway's Game of Life is implemented on a regular lattice with Moore neighborhoods and periodic boundary condition [20]. This classic cellular automata model can produce complex behaviors and emergence. Initially, the lattice is randomly populated with a given target density, usually between 20-40%. Our simulations set this value at 30%. At each time step, nodes are updated synchronously. For each node, one of three possible actions occurs: (1) birth - a dead node with exactly three neighbors becomes a live node; (2) survival - a living node with 2-3 neighbors continues to survive; and (3) death - a live node with less than 2 or more than 3 neighbors dies. This process continues until a stable condition is reached, usually when a mass-extinction results in a density too low to maintain interactivity.

4.2.2 Results

Figure 3 shows high levels of complexity within the initial time window with mostly decreasing complexity throughout the simulations. This is consistent with our understanding of the Game of Life. Due to the strength of its rules and above-equilibrium density present at initialization, large die-off events are likely, and interactivity is expected to diminish. The higher level of Jensen-Shannon divergence observed per transfer entropy in the initial phase appears to indicate strong sensitivity to interactivity which is consistent with Game of Life model behavior.

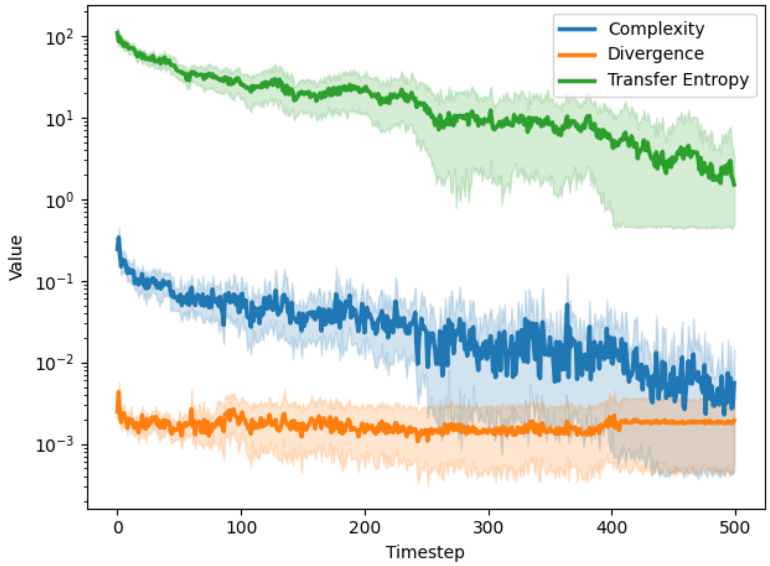

Figure 3: Jensen-Shannon Divergence, Transfer Entropy, and Complexity results for Game of Life model simulations.

4.3 Susceptible-Infected-Susceptible

4.3.1 Model

Susceptible-Infected-Susceptible models are commonplace in epidemiology. This model was similarly initialized on a regular lattice with Moore neighborhoods and a periodic boundary condition. The grid is fully populated by individuals: 15% are infected and the remaining 85% are susceptible. Of those infected, the infected time period is randomly set with between 1-3 time steps remaining. At each time step, nodes are updated synchronously. Each node with infected neighbors has an additional 10% chance to become infected per each infected neighbor. Once infected, an individual will remain infectious for 3 time steps. After this period, the individual will become susceptible again.

4.3.2 Results

The most dramatic results were obtained from the SIS model simulations. Figure 4 shows high levels of Jensen-Shannon divergence, transfer entropy, and complexity as the disease spreads initially through the population. Afterward, high levels of transfer entropy are maintained as waves of susceptibility, infection, and immunity cycle through the population. Jensen-Shannon divergence diminishes as the relative proportions of infected and uninfected individuals in the population stabilize.

However, it is important to note that divergence is still observed as these proportions continue to fluctuate albeit at a lesser rate. This example also illustrated the case that a system's change in entropy, ΔH over time can be substantially different from its Jensen-Shannon divergence, see Figure 5. Change in entropy, the difference in entropy between two time steps, indicates a change in the heterogeneity of the system's states. We may observe dynamism by computing this difference. Negative ΔH was frequently and observed for these simulations. Due to this, $\sum_t \Delta H$ yields starkly different results. One could correct for this by employing $\sum_t |\Delta H|$. However, no correction is available to address the pitfall discussed in 3.2. Namely, a real change in the distribution of states may not result in a change in entropy if heterogeneity in states is constant. This is a fundamental limitation of Shannon entropy as a measure of dynamism.

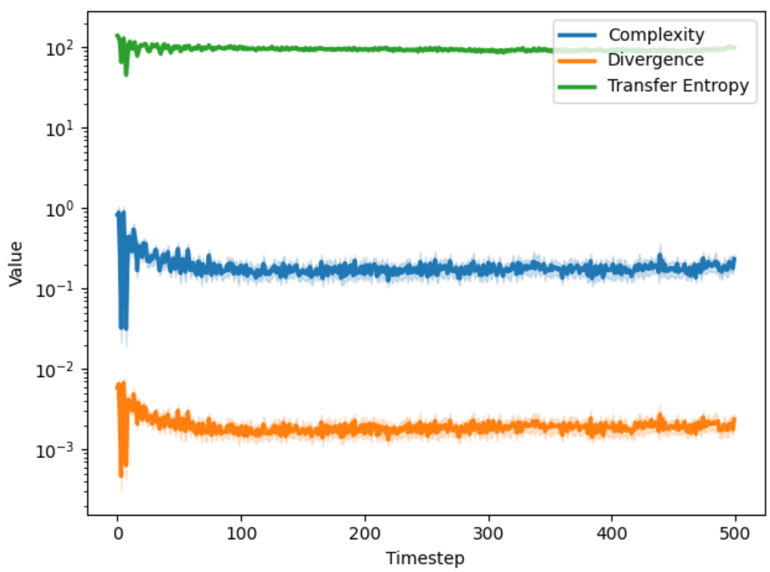

Figure 4: Jensen-Shannon Divergence, Transfer Entropy, and Complexity results for Susceptible-Infected-Susceptible model simulations.

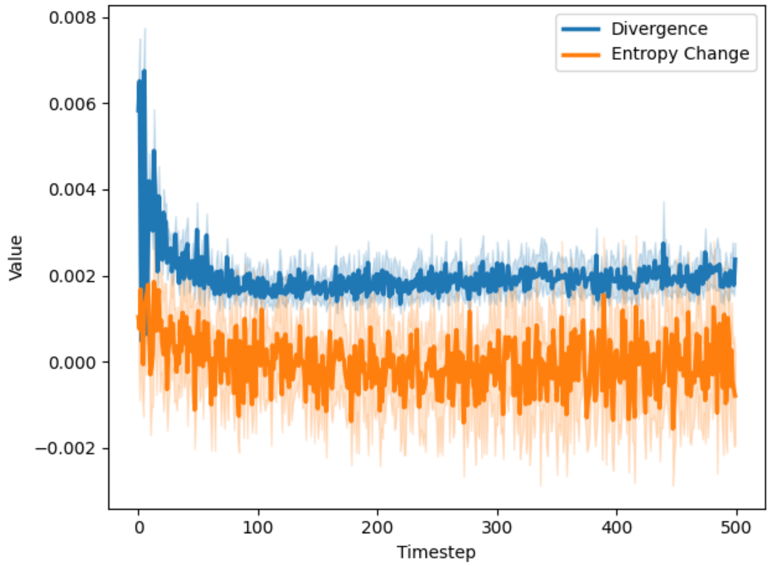

Figure 5: Jensen-Shannon divergence and entropy results for the SIS model simulations.

5 Discussion

Figure 6 shows the varied complexity trajectories for each of the systems investigated. The SIS model showed the greatest and most persistent levels of complexity. The Game of Life model displayed decreasing levels of complexity. The Xie & Zhou model also displayed persistent complexity, but at lower levels. The Schelling model displayed complexity similar to the Xie & Zhou model before collapsing earliest at around $t = 200$.

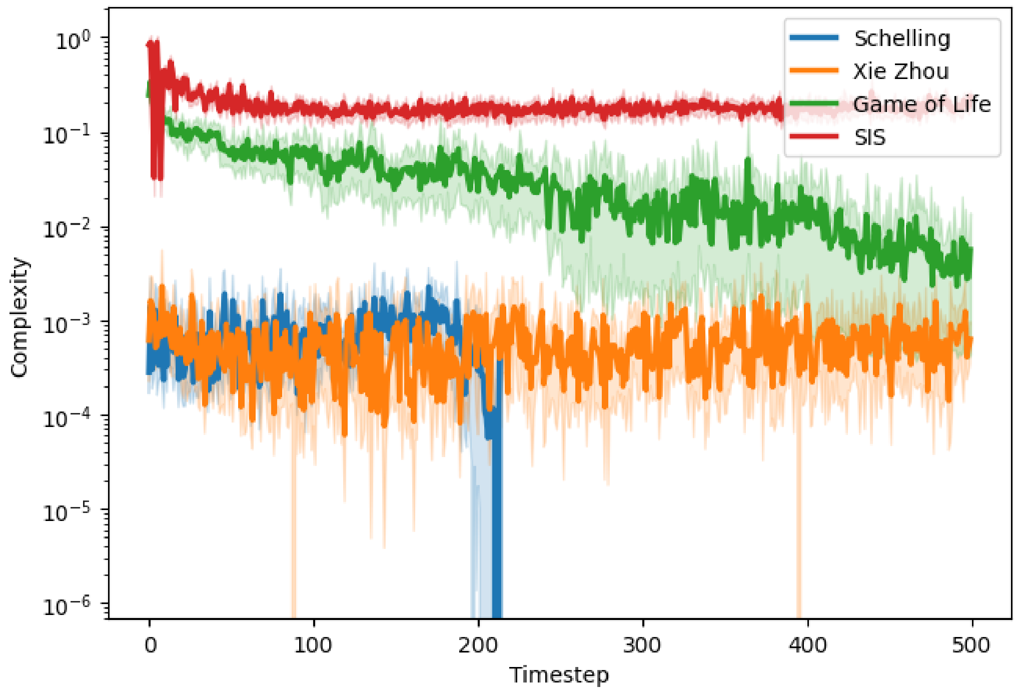

Figure 6: Complexity results for all model simulations.

The proposed measure of complexity performed as expected in each of these conditions by reflecting the intersection of interactivity and dynamism amongst their constituent components. The results from the segregation models were consistent with Weaver's [1] suggestion that the addition of stochastic behavior to the elements of a system increase its complexity. Puig et al. [21] have shown the heterogeneity likewise extends criticality. Similarly, we have shown preliminarily that heterogeneity extends complexity. The underpinning concept here may be that both complexity and criticality require variety as described by Ashby [22].

The results from the Game of Life and SIS models illustrated two key phenomena: (1) Jensen-Shannon divergence captures dynamism in systems where Shannon entropy may mislead, and (2) dynamism may occur without meaningful interactivity. Together, the results demonstrate the complementary value of the proposed measure. It enables discussion of how much complexing a system did during a given Δt. Conversely, examining the Shannon entropy for the same Δt would only enable discussion of the heterogeneity of its states for that period. This is because Shannon entropy alone does not ensure that dynamism was attributable to interaction by the system's elements.

The primary limitation of this work arises from the computational complexity inherent in the formulation of the measure. Future work is required to develop a more comprehensive methodology for computation of this measure for a wider variety of system descriptions.

An attractive avenue for future investigation is cross-scale complexity. By selecting processes at both higher and lower scales of system, and noting that transfer entropy is asymmetrical, we could examine interactive complexity bidirectionally. Here we could elaborate Gershenson's [3]'s assertion that both upward and downward emergence may occur.

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