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DYNAMIC MODELS OF ECONOMIC DEVELOPMENT
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The following approval page bearing the signature of the chairman and all members of the student's committee and the chairman of the department occupies the page as follows:

DYNAMIC MODELS OF ECONOMIC DEVELOPMENT

AND INTERNATIONAL CAPITAL MOVEMENTS

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AND INTERNATIONAL CAPITAL MOVEMENTS

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PREFACE

There has been a growing awareness that the traditional theory of international capital movements is static and must be reformulated by including dynamic elements that are common in the actual world. This awareness has long since occupied the mind of the present author whose study covers not only the so-called 'classical' theory of J. S. Mill, F. W. Taussig, F. G. Graham, *et al.*, and the 'modern' theory of B. Ohlin, G. Harberler, R. F. Harrod, *et al.*, in the field of international economics, but also the postwar development of the theory of economic growth initiated by R. F. Harrod and E. D. Domar. The lack of dynamic factors in the traditional theory of capital movements is due to the fact that capital movements have almost always been considered as a short-run problem that could and should be solved within the established balance-of-payments mechanism. The static and short-run nature of the established theory needs now to be supplemented by consideration of the dynamic and long-run aspects of international capital movements observed in the real world. The role of capital as a real factor of production ought to be introduced explicitly. Only fragments of that type of capital movement theory which is concerned with real capital as a productive factor exist today, as has been pointed out by R. Nurkse. In short, it is of academic as well as practical importance to theorize the dynamic and long-run aspect of international capital movements.

When it comes to the theory of economic growth, thus far there have been no substantial attempts made in this direction with the

exception of a few such as H. G. Johnson's, H. Brems', P. K. Bardhan's and D. C. Smith's. Among the two major currents of growth theory, the Harrod-Domar-type fixed-coefficient model still retains usefulness in comparison with the neo-classical variable-coefficient model. For the actual world is composed of capital-deficient economies, on the one hand, and economies with labor unions and pressure groups, on the other.

This work has three analytical purposes: The _first_ is to reveal possible effects of capital imports on the rapid growth of an underdeveloped economy, the _second_ is to indicate possible effects of capital exports on the stable growth of an advanced economy, and the _third_ is to bring out the interacting effects of international capital movements on the respective growth patterns of the two economies in the mixed advanced-underdeveloped world. For the purposes of analysis, the original Harrod-Domar growth model for a closed economy shall be extended and transformed into an explicitly dynamic model for an _open_ economy. The mathematical model built therefor will constitute the core of the present attempt to analyze the international capital movements in the context of a growing world economy.

The whole edifice is divided into nine components. The first chapter presents the derivation and discussion of the fundamental growth model for an open economy, which is applicable to any of the economies, advanced or underdeveloped. Brief reference will be made to the disaggregation problem of the macro growth equations in the Appendix to Chapter 1. The second and third chapters are designed for an underdeveloped economy with potential rapid growth. Chapter 2 offers an

application of the fundamental model of Chapter 1 to the underdeveloped economy with the role of capital imports into explicit account. A disaggregated model of Chapter 2 leads to optimizing models of Chapter 3, into which capital imports are incorporated. By way of comparison, the neoclassical efficient growth model applied to such an open economy will be presented in the Appendix to Chapter 3. The fourth and fifth chapters deal in turn with the stable growth of an advanced economy. Chapter 4 presents an endogenous open-economy model of cyclical growth with a view to analyzing countercyclical capital exports for stable growth. This is followed by the programming models of Chapter 5, which involve another application of the technique developed in Chapter 2 to an advanced economy engaged in capital exports. The subsequent three chapters synthesize the preceding single-country models. Chapter 6 investigates dynamic interactions of a developing underdeveloped and a growing advanced economy, and it reveals dynamic effects of exports and capital movements on the respective growth patterns. Chapter 7 offers programming models for narrowing the gap between the advanced and the underdeveloped economy. This is followed by the monetary analysis of Chapter 8, in which long-run trends of the balance of payments of the respective economies will be discussed. All of these chapters culminate in Chapter 9 which presents the conclusions of the present study.

Methodologically speaking, the present work might be regarded as belonging to the area of mathematical economics. It is true that the mathematical approach makes it possible to implement economic analysis

in rigorous terms. However, it is far from the author's intention to put the cart of mathematics before the horse of economics. The use of mathematics in the present study shall remain auxiliary to the substance of economic thinking.

The present study handles foreign-aid-type capital movements with no interest payments partly because foreign aid looms very large in international capital movements between advanced and underdeveloped economies[1] and partly because it simplifies analysis. The main reason for the neglect of the role of interest is that interest rates are in reality not necessarily higher in capital-poor underdeveloped economies than in capital-rich developed economies and that profit-seeking capitals do not always flow from the latter into the former.[2] Any formal attempt to study the self-help problem of underdeveloped economies and hence the relationship between foreign aid and mobilization of domestic savings[3] lies beyond the scope of the present work. No attention is paid to structural changes due to capital imports on the part of developing countries.[4]

Among principal works which motivated the present attempt were the

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[1] Bela Balassa, "The Capital Needs of the Developing Countries," Kyklos, Vol. XVII, 1964, p. 203.

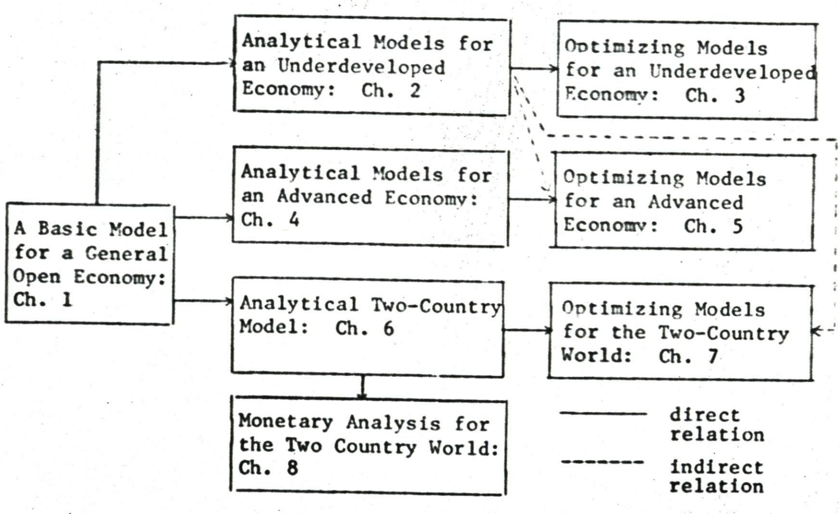
[2] H. W. Arndt, "A Suggestion for Simplifying the Theory of International Capital Movements," Economia Internationale, August 1956, p. 476.

[3] John C. H. Fei and Douglas S. Paauw, "Foreign Assistance and Self-Help: A Reappraisal of Development Finance," The Review of Economics and Statistics, Vol. XLVII, 1965, pp. 251-267.

[4] Fanny Ginor, "The Impact of Capital Imports on the Structure of Developing Countries," Kyklos, Vol. XXII, 1969, pp. 104-123.

following three papers: Prof. Harry G. Johnson's 'Equilibrium Growth in an International Economy,' which gave an initial stimulus to the present inquiry; Prof. Henri Theil's 'International Inequalities and General Criteria for Development Aid,' which provided the idea of juxtaposing an advanced and an underdeveloped economy; and Prof. Kenneth K. Kurihara's 'Linear Programming for Multisectoral Optimal Growth,' which supplied the conception of multisectorizing Harrod's aggregate growth model.

The organization of the whole study can be illustrated as follows:



CHAPTER I

A DYNAMIC HARROD-DOMAR GROWTH MODEL FOR AN OPEN ECONOMY

Section 1

Domestic Equilibrium Growth

Domestic equilibrium growth is here defined as the condition which realizes and maintains full utilization of existing capital and, under the existing technology and the given labor force, the maximum feasible level of product. Domestic equilibrium growth thus guarantees contentment on the side of entreprepneurs and the highest possible income over a long period; in Harrod's terminology it implies $G_w = G_n$, so that, if the actual growth rate is equal to it, i.e. $G = G_w = G_n$, neither any unstable variations nor any secular stagnation and chronic inflation would take place. Naturally the present concept of domestic equilibrium involves the technical relation, the persistent equality of the saving-side to the investment-side in the open economy.[1] The adjective 'domestic' is added to distinguish it from 'external' equilibrium to be defined later.

The model to be presented is dynamic in the sense that it consists of functions of time. The author intends to build it in a continuous, rather than a discrete, form of time because economic variables change

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[1]Domestic equilibrium growth is equivalent to achieving a growth path consistent with potential output.

continuously in value over time. It is also dynamic in Harrod's terminology that a rate of growth appears as an unknown variable.[2]

Notations:

Y : net national product or output

$Y(0)$: its initial value

C : domestic consumption

I : domestic investment

$\bar{I}$: autonomous investment

S : domestic saving

X : exports of goods and services

$\bar{X}$: autonomous exports; $\bar{X}(0)$, its initial value

M : imports of goods and services

B : the balance of payments on current account

c : marginal propensity to consume

$\bar{c}$: autonomous consumption

m : marginal propensity to import

$\bar{m}$: autonomous imports

λ : growth rate of the autonomous exports

ρ : marginal capital coefficient

p_x : price index of the exports

p_m : price index of the imports

t : time

ϵ : marginal propensity to export

ϕ : terms of trade

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[2] R. H. Harrod, 'An Essay in Dynamic Theory,' Economic Journal, March 1939, p. 17.

Autonomous variables mean here those influenced by non-output factors, such as technology, policy, social organizations, etc.

In order for the open economy to attain capacity output, the sum of domestic investment and exports must be equal to the sum of domestic saving and imports. Since saving is that part of national income which is not consumed, the ex-ante requirement of domestic equilibrium is expressed by

$$I = S + M - X: I + X = S + M$$

where

$$S = Y - C.$$

Thus the ex-ante requirement of domestic equilibrium is expressed by

$$Y = C + I + X - M. \qquad (1\text{-}1\text{-}1)$$

The domestic consumption is assumed to consist of autonomous as well as induced consumption expenditures, i.e.

$$C = \bar{c} + cY \qquad (1\text{-}1\text{-}2)$$ [3]

In view of the universal need for social overhead capital in almost all economies, advanced or underdeveloped, we take into explicit account the autonomous investment which plays a secondary role next to the acceleration principle in the original Harrod-Domar model. Thus the present investment function consists of the autonomous as well as the output-induced investment

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[3]To be exact, (1-1-2) should be written as: $C = \bar{c} + cY + u_c$. u_c is the random element involved in the linearity, which is to be explained in the Appendix to Ch. 1. All that is needed here is the assumed normal distribution of u_c. Similar considerations apply to other linear relations. See p. 26.

$$I = \rho\dot{Y} + \overline{I} . \qquad (1\text{-}1\text{-}3)$$

For the sake of convenience, autonomous investment is assumed to be independent of time. And investment is considered net of depreciation, whether it is autonomous or induced.

Though exports are usually treated as an autonomous variable in the income analysis, the exports function here consists not only of this variable but also of a function of the level of output

$$X = \overline{X} + \epsilon Y = \overline{X}(0)\, e^{\lambda t} + \epsilon Y . \qquad (1\text{-}1\text{-}4)$$

Output is considered as a rough indicator of an open economy's stage of industrialization affecting its export capacity. The higher the level of output, the more developed is the economy and hence, generally speaking, the greater is its competitive power in the exports market. The relation between the level of output and the exporting capacity, which is symbolized by ϵ, varies from country to country.

$$M = \overline{m} + mY . \qquad (1\text{-}1\text{-}5)$$

The balance of payments on the current account (the income account) is the difference between the value of exports and that of imports:

$$B = p_x X - p_m M , \qquad (1\text{-}1\text{-}6)$$

where factor incomes and unilateral transfers are assumed away.

From (1-1-1), (1-1-2), (1-1-3), (1-1-4) and (1-1-5) one can easily solve the rate of growth of the net national product required for the domestic equilibrium growth rate denoted by G_e :

$$G_e = \frac{\dot{Y}}{Y} = \frac{1}{\rho}\left[h - \frac{\overline{X}(0)\, e^{\lambda t} - h R_0}{Y}\right] = \frac{1}{\rho}\left[-\frac{X - M}{Y} + \frac{S}{Y} - \frac{\overline{I}}{Y}\right] \qquad (1\text{-}1\text{-}7)$$

where $h = 1 - c + m - \epsilon = MPS + m - \epsilon$ and $R_0 = \frac{\bar{m} - \bar{c} - \bar{I}}{h}$. This means that if the proportional export surplus falls over time, the growth rate of domestic-equilibrium output rises over time, and vice versa. Implied in the ideal growth rate G_e, (1-1-7) is, by definition, the simultaneous realization of G_w and G_n in Harrod's terms, i.e.

$$G_e(1\text{-}1\text{-}7) = G_w(\text{Harrod}) = G_n(\text{Harrod}).$$

In order to know how the level of output is determined we proceed to solve the differential equation (1-1-7). The solution, the level of productive capacity required for the domestic equilibrium growth of the open economy denoted by Y_e, is given by

$$\text{(i)} \quad \lambda \neq \frac{h}{\rho}, \quad Y_e = \frac{\bar{X}(0)}{h - \lambda\rho} e^{\lambda t} + \left[Y(0) - \frac{\bar{X}(0)}{h - \lambda\rho} + R_0 \right] e^{\frac{h}{\rho} t} - R_0$$

$$\text{(ii)} \quad \lambda = \frac{h}{\rho}, \quad Y_e = -\frac{\bar{X}(0)}{\rho} t e^{\lambda t} + \left[Y(0) + R_0 \right] e^{\lambda t} - R_o . \qquad (1\text{-}1\text{-}8)$$

In substituting (1-1-8) for Y in the rightmost side of (1-1-7), one can enumerate all the factors influencing the growth rate and the level of net national output required for the domestic equilibrium of the open economy: the exogenous exports (its initial value and its growth rate) and the other strategic parameters (marginal capital-output ratio, marginal propensity to consume, marginal propensity to import, marginal propensity to export, autonomous imports, autonomous consumption, and, last but not least, autonomous investment). The respective relation of one or some of these parameters to the equilibrium output

Y_e and its growth rate can be indicated by partial derivatives. Though the marginal relation of Y_e to h has turned out to be too complicated to allow meaningful interpretations, the partial derivatives of Y_e and G_e, with respect to R_0, are simple enough, to wit

$$\text{(i)} \quad \frac{\partial Y_e}{\partial R_0} = e^{\frac{h}{\rho} t} - 1 > 0 \; ;$$

$$\frac{\partial G_e}{\partial R_0} = \frac{1}{\rho}\left(\frac{\bar{X}(0)\; e^{\lambda t} - h\; R_0}{Y^2}\right)\left(e^{\frac{h}{\rho} t} - 1\right) > 0 \quad \text{after} \quad t'$$

$$\text{(ii)} \quad \frac{\partial Y_e}{\partial R_0} = e^{\lambda t} - 1 > 0$$

$$\frac{\partial G_e}{\partial R_0} = \frac{1}{\rho}\left(\frac{\bar{X}(0)\; e^{\lambda t} - h\; R_0}{Y^2}\right)\left(e^{\lambda t} - 1\right) > 0 \quad \text{after} \quad t'$$

$$\left[\; t' = \frac{1}{\lambda} \log\left(\frac{h\; R_0}{\bar{X}(0)}\right) \;\right]$$

That is, a change in any of autonomous variables resulting in an increment of R_0 has, cet. par., proved to enhance, in the long run, the equilibrium output (1-1-8) and its growth rate. This result may give an unusual impression in that an increased autonomous import, a decreased autonomous consumption and/or a decreased autonomous investment serve to raise the (potential) output and its growth rate. But the reader familiar with the multiplier theory is warned that the present study does not concentrate on the demand side(see p.10).

The marginal effect of the growth rate of autonomous exports on

the equilibrium output (1-1-8) can be measured by

$$\text{(i)} \quad \frac{\partial Y_e}{\partial \lambda} = \frac{\bar{X}(0)}{(h-\lambda\rho)^2}\left[\rho\left(e^{\lambda t} - e^{\frac{h}{\rho}t}\right) + t(h-\lambda\rho)\right]$$

$$\frac{\partial G_e}{\partial \lambda} = \left(\frac{\bar{X}(0)\ e^{\lambda t} - h\ R_0}{Y^2}\right)\left[\rho\left(e^{\lambda t} - e^{\frac{h}{\rho}t}\right) + t(h-\lambda\rho)\right]$$

$$\text{(ii)} \quad \frac{\partial Y_e}{\partial \lambda} = \left(-\frac{\bar{X}(0)}{\rho}t + Y(0) + R_0\right) t\, e^{\lambda t}$$

$$\frac{\partial G_e}{\partial \lambda} = \frac{1}{\rho}\left(\frac{\bar{X}(0)\ e^{\lambda t} - h\ R_0}{Y^2}\right)\left(-\frac{\bar{X}(0)}{\rho}t + Y(0) + R_0\right) t\, e^{\lambda t}$$

This leads us to the following illustration.

Table 1-1-1

The Relation of the Domestic-Equilibrium Output (1-1-8) and its Growth Rate (1-1-7) to the Growth Rate of Exports

$\lambda < \frac{h}{\rho}$	$\lambda = \frac{h}{\rho}$	$\lambda > \frac{h}{\rho}$
$\frac{\partial Y_e}{\partial \lambda} < 0$ after t^*	$\frac{\partial Y_e}{\partial \lambda} < 0$ after t^{**}	$\frac{\partial Y_e}{\partial \lambda} > 0$
$\frac{\partial G_e}{\partial \lambda} < 0$ after t^*	$\frac{\partial G_e}{\partial \lambda} < 0$ after t^{**}	$\frac{\partial G_e}{\partial \lambda} > 0$ after t^*

t^* is the larger of the two points t' and t^{Δ}, $(h - \lambda\rho)t^{\Delta} + \rho(e^{\lambda t^{\Delta}} - e^{(h/\rho)t^{\Delta}}) = 0$, i.e. $(1/\lambda)\log(h\, R_0/X(0)) = t' \lesseqqgtr t^* \gtreqqless t^{\Delta}$. t^{**} is the larger of the two points t' and $\rho(Y(0) + R_0)/X(0)$, i.e. $\rho(Y(0) + R_0)/X(0) \lesseqqgtr t^{**} \gtreqqless t'$. Differentiating (1-1-1) with respect to Y yields $1 = c + (d\,I/d\,Y) + \epsilon - m$. $d\,I/d\,Y = 1 - c + m - \epsilon \equiv h$. Hence $h/\rho = (d\,I/d\,Y)/\rho = (d\,I/d\,Y)/(I/dY) = d\,I/I$ (growth rate of investment). Thus the growth rate of exports exceeding the one of investment tends to enhance the equilibrium output and its growth rate, and vice versa. This growth rate of investment h/ρ is an increasing function of the marginal propensity to save and to import and marginal capital productivity.

The equilibrium growth rate G_e given by (1-1-7) possesses the following characteristic (cf. Table 1-3-1):

Table 1-1-2

The Relation between the Growth Rate of Domestic-Equilibrium Output and that of Exports

$\lambda < \tau_0$	$\lambda > \tau_0$
$G_e > 0$	$G_e < 0$

$$\tau_0 = \frac{1}{\rho}\left[h - \frac{X(0)}{Y(0) - R_0}\right] \lesseqqgtr G_e(0)$$

If R_0 is zero, the value τ_0 is equal to the initial growth rate of the equilibrium output; in this case the result means that the growth rate of exports beyond that of the equilibrium output at the initial

point induces the latter to fall over time, and vice versa. Explicit differentiation of G_e with respect to time results in

$$G_e = -(X/\rho Y)(\lambda - G_e),$$

which means that if the growth rate of exports exceeds that of output, the latter decreases over time, and vice versa. This is the conclusion originally stated succinctly by R. F. Harrod[4] and further explicated by H. G. Johnson[5].

When capital movements enter the picture, the domestic equilibrium growth rate (1-1-7) changes to

$$G_e = \frac{1}{\rho}\left[h - \frac{\dot{X}(0)\ e^{\lambda t} - h\ R_0 - K_F}{Y}\right], \qquad (1\text{-}1\text{-}7)'$$

where $K_F > 0$ means net capital imports (saving from abroad) and $K_F < 0$ net capital exports (investment abroad). Thus the larger the capital imports, the larger tends to become the growth rate (1-1-7)'[6]. The larger the capital exports, the smaller tends to become the growth rate. An economy which aims at a rapid growth should receive more capital. An economy whose objective is to realize a relatively low and stable rate of growth can resort to capital exports. These aspects will be more fully discussed in subsequent chapters.

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[4] Towards a Dynamic Economics, London, Macmillan, 1951, p. 107.

[5] International Trade and Economic Growth, London, Allen & Unwin, 1958, pp. 122-123.

[6] This view corresponds to what R. J. Ball calls the 'orthodox' view on capital imports and economic development. 'Capital Imports and Economic Development: Paradoxy or Orthodoxy?', Kyklos, No. 3, 1962.

If the reader thinks that the relation between foreign trade and capacity growth is exclusively analyzed here, it may be appropriate to point out the relation between foreign trade and demand growth which is involved in the present model. This induces us to regard the supply side of investment as given, in other words, $\rho = 0$ (zero capital coefficient, or no acceleration principle). Under this condition it is not hard to derive from (1-1-1), (1-1-2), (1-1-3), (1-1-4) and (1-1-5) the following formula

$$Y_e = \frac{1}{(1 - c + m - \epsilon)}(X - \overline{m} + \overline{c} + \overline{I}) \qquad (1\text{-}1\text{-}9)$$

which is nothing but a familiar Keynesian foreign trade multiplier formula.

Comparison of the equation (1-1-8) with the one (1-1-9) makes it possible to discuss contradictory effects of foreign trade variables on the domestic equilibrium output and the effective demand. The first thing to notice is that, on the demand side, the larger the marginal propensity to import m and the autonomous imports $\overline{m}$, the smaller becomes, cet. par., the effective demand, whereas the larger these import variables are, the larger tends to be the domestic equilibrium output (1-1-8). Second, it is known from the model that the larger the export variables (that is, the growth rate of the exports and the marginal propensity to export) are, the more inflationary pressure is cet. par. expected on the demand side (1-1-9), while from the angle of domestic equilibrium growth, the larger these export variables are, the smaller tends to become the domestic equilibrium output (1-1-8).

Section 2

External Equilibrium Growth

Compared with domestic equilibrium growth it is easy to analyze external equilibrium growth. External equilibrium here means simply a zero difference between exports and imports on current account; external equilibrium growth corresponds to the growth of output satisfying it continuously. The external equilibrium thus defined reduces itself to

$$B = p_x X - p_m M = 0 . \qquad (1\text{-}1\text{-}6)'$$

Substituting (1-1-4) and (1-1-5) for the corresponding terms in (1-1-6)', we get the time path of the external equilibrium growth of output denoted by Y_f :

$$Y_f = \phi \frac{\bar{X}(0)}{m - \phi\epsilon} e^{\ t} - \frac{\bar{m}}{m - \phi\epsilon} \qquad (1\text{-}2\text{-}1)$$

where $m - \phi\epsilon \neq 0$ (assumed). This output path satisfies external equilibrium at any moment of time, regardless of whether domestic equilibrium is realized or not.

Section 3

Domestic and External Equilibrium Growth

Classical economists tended to stress the need for international equilibrium inasmuch as Say's Law was believed to work in the national

economy, while Keynesian economists are in turn inclined to stress the need for domestic equilibrium. The growth path which brings a simultaneous realization of both domestic and international equilibria, i.e. $G = G_n = G_w$, may be termed, to borrow Joan Robinson's terminology, The "Golden-Age" Path. Although it is extremely difficult for an actual economy to be in equilibrium both domestically and externally, it is still worthwhile to make an attempt to clarify the condition for both equilibria[7]. For this purpose let us first examine the relation between the domestic equilibrium growth of output (1-1-8) and the external equilibrium growth of output (1-2-1). The two factors, the rate of growth of exports and the net terms of trade, have proved to play an important role, as shown below:

[7]A similar attempt is aimed at by Y. W. Swan, 'Economic Control in a Dependent Economy,' Economic Record, March 1960, pp. 51-66. His policy-oriented static study explores the condition under which 'Internal Price Stability' as well as 'Internal Balance' (full employment without inflation) and External Balance (balance of payments equilibrium) come true to conclude that 'Internal and External Balance' are realized at a point representing a certain combination of aggregate domestic demand, money wage level and external price level. No explanation has been given as to how the definite figures corresponding to the three objectives are derived and connected with each other though.

Table 1-3-1

The Relation between Domestic Equilibrium and External Equilibrium

Y_e: the output realizing full utilization of capital

Y_f: the output maintaining zero balance of payments on current account

t°: the time at which Y_e and Y_f intersect with each other

$\bar{X}(0) > 0, \quad Y(0) > 0, \quad h = 1 - c + m - \epsilon, \quad R_0 = (\bar{m} - \bar{c} - \bar{I})/h,$

$$\tau_0 = \frac{1}{\rho}\left[h - \frac{\bar{X}(0)}{Y(0) - R_0}\right], \quad \frac{m - \phi\epsilon}{h - \lambda\rho} = \gamma, \quad G_f = \frac{\dot{Y}_f}{Y_f}, \quad G_e = \frac{\dot{Y}_e}{Y_e},$$

$$q = \left[Y(0) + \frac{\bar{m}}{m}\right]\frac{m}{\bar{X}(0) + \epsilon Y(0)}, \quad \phi = \frac{p_x}{p_m}.$$

<table>
<tr><td colspan="4">c $\lambda < h/\rho$</td><td rowspan="3">b
$\lambda \geqq \frac{h}{\rho}$</td><td rowspan="3"></td></tr>
<tr><td rowspan="2">c-3
$\lambda < \tau_0$</td><td colspan="2">c-2
$\lambda = \tau_0$</td><td rowspan="2">c-1
$\lambda > \tau_0$</td></tr>
<tr><td>c-2-1
$\gamma \geqq \phi$</td><td>c-2-2
$\gamma < \phi$</td></tr>
<tr><td>$Y_f > Y_e$ up to t°;
$Y_e > Y_f$ after t°.
$\dot{Y}_e > \dot{Y}_f > 0$
$G_e > G_f > 0$</td><td>$Y_f > Y_e$ up to t°;
$Y_e > Y_f$ after t°.
$\dot{Y}_e > \dot{Y}_f > 0$
$G_e > G_f > 0$</td><td>$Y_f > Y_e$
$\dot{Y}_f > \dot{Y}_e > 0$
G_e ? G_f</td><td colspan="2">$Y_f > Y_e$
$\dot{Y}_f > 0 > \dot{Y}_e$
$G_f > 0 > G_e$</td><td>a-1
$\phi > q$</td></tr>
<tr><td>$Y_e > Y_f$
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$\dot{Y}_e > \dot{Y}_f > 0$
G_e ? G_f</td><td>$Y_e > Y_f$ up to t°;
$\dot{Y}_f > \dot{Y}_e > 0$
$G_f > G_e > 0$</td><td colspan="2">$Y_e > Y_f$ up to t°;
$\dot{Y}_f > 0 > \dot{Y}_e$
$G_f > 0 > G_e$</td><td>a-2
$\phi \leqq q$</td></tr>
</table>

Table 1-3-1 is comprehensive but not necessarily convenient to allow meaningful interpretations. For this purpose the following table is added under simplifying assumptions:

Table 1-3-2

The Relation between Domestic Equilibrium and External Equilibrium under the simplifying assumptions: $\bar{c} = \bar{m} = \bar{I} = 0$ and $t > t°$

<table>
<tr><td colspan="4">c $\lambda < h/\rho$</td><td rowspan="3">b
$\lambda \geqq \frac{h}{\rho}$</td><td rowspan="3"></td></tr>
<tr><td rowspan="2">c-3
$\lambda < \tau_0$</td><td colspan="2">c-2
$\lambda = \tau_0$</td><td rowspan="2">c-1
$\lambda > \tau_0$</td></tr>
<tr><td>c-2-1
$\gamma \geqq \phi$</td><td>c-2-2
$\gamma < \phi$</td></tr>
<tr><td colspan="2" rowspan="2">$Y_e > Y_f$
$G_e > G_f > 0$</td><td colspan="3">$Y_f > Y_e$</td><td rowspan="2">a-1
$\phi > q$</td></tr>
<tr><td>$G_f > G_e > 0$</td><td colspan="2">$G_f > 0 > G_e$</td></tr>
<tr><td colspan="2" rowspan="2">$Y_e \geqq Y_f$
$G_e > G_f > 0$</td><td colspan="3">$Y_f > Y_e$</td><td rowspan="2">a-2
$\phi \leqq q$</td></tr>
<tr><td>$G_f > G_e > 0$</td><td colspan="2">$G_f > 0 > G_e$</td></tr>
</table>

Table 1-3-1 or Table 1-3-2 can give us the necessary and sufficient conditions for maintenance of both domestic and external equilibria:

$$\lambda = \tau_0 \quad \text{and} \quad \phi = \gamma = q \, . \tag{1-3-1}$$

This is indeed a tall order. Only God seems to know how to adjust the potential policy variables of the model, so that λ is set equal to τ_0 and simultaneously both γ and q are set equal to the exogenous terms of trade ϕ .

Table 1-3-1 suggests that a sufficiently high growth rate of exports $(\lambda > \tau_0)$ serves to make the growth rate of output (1-1-7) negative, one of the reasons being its deflationary pressure on the domestic productive capacity[8]. On the other hand, a constant upward shift of output is expected so long as the growth rate of exports remains within a certain limit $\lambda \leqq \tau_0$. In this connection it is interesting to note that the rate of growth of exports of Japan in the 1950's, though relatively high compared with other countries, was not so high as to exceed excessively its growth rate of net national output. The case of $\lambda \leqq \tau_0$ (c-3 and c-2-1) is exemplified by the one in which we can take the conditions of supply as given, that is $\rho = 0$. Thus Table 1-3-1 or its variant Table 1-3-2 has turned out to comprise as its special case the effect of demand growth on the level and the growth rate of net national output which is usually handled by the foreign trade multiplier analysis.

On the assumption that the domestic equilibrium condition is met, we can next show how the balance-of-payments position varies in accordance with the two key parameters, the growth rate of exports and the net terms of trade. The result is condensed in Table 1-3-3.

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[8] See e.g. Kenneth K. Kurihara, The Keynesian Theory of Economic Development, Columbia University Press, 1959, p. 38.

Table 1-3-3

The Balance of Payments on Current Account, B ,
when the domestic equilibrium is realized continuously

<table>
<tr><td colspan="4">c
$\lambda < h/\rho$</td><td rowspan="3">b
$\lambda \geqq \frac{h}{\rho}$</td><td rowspan="3"></td></tr>
<tr><td rowspan="2">c-3
$\lambda < \tau_0$</td><td colspan="2">c-2 $\lambda = \tau_0$</td><td rowspan="2">c-1
$\lambda > \tau_0$</td></tr>
<tr><td>c-2-1
$\gamma \geqq \phi$</td><td>c-2-2
$\gamma < \phi$</td></tr>
<tr><td colspan="2">$B > 0$ up to $t°$;
$B < 0$ after $t°$.
$\dot{B} < 0$</td><td colspan="3">$B > 0$
$\dot{B} > 0$</td><td>a-1
$\phi > q$</td></tr>
<tr><td>$B < 0$
$\dot{B} < 0$</td><td>$B \leqq 0$
$\dot{B} \leqq 0$</td><td colspan="3">$B < 0$ up to $t°$;
$B > 0$ after $t°$.
$\dot{B} > 0$</td><td>a-2
$\phi \leqq q$</td></tr>
</table>

A diagrammatic illustration of Table 1-3-3 can be given in the figures on the next page.

Figure 1-3

The Balance of Payments on Current Account Illustrated

+ means surplus, $B > 0$; - means defict, $B < 0$

Figure 1-3-1: $q > \gamma$

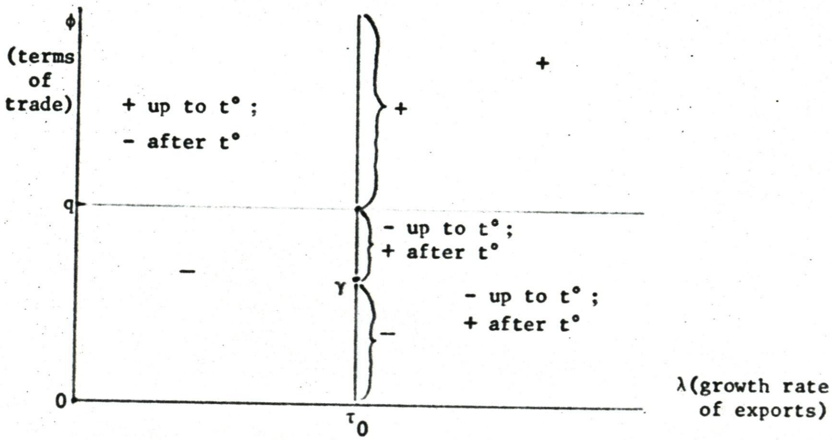

Figure 1-3-2: $\gamma > q$

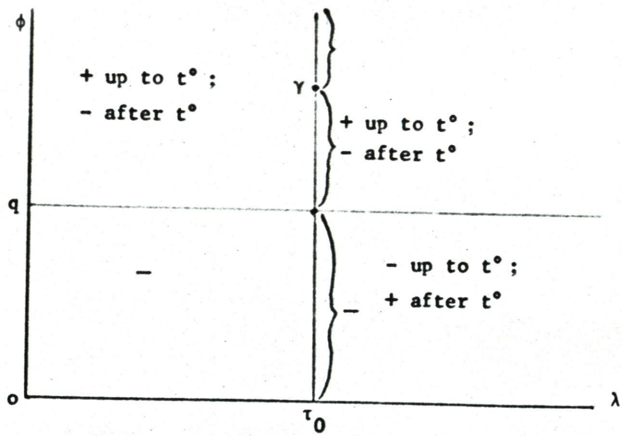

Figure 1-3-3: $q = \gamma$ (ideal but exceptional case)

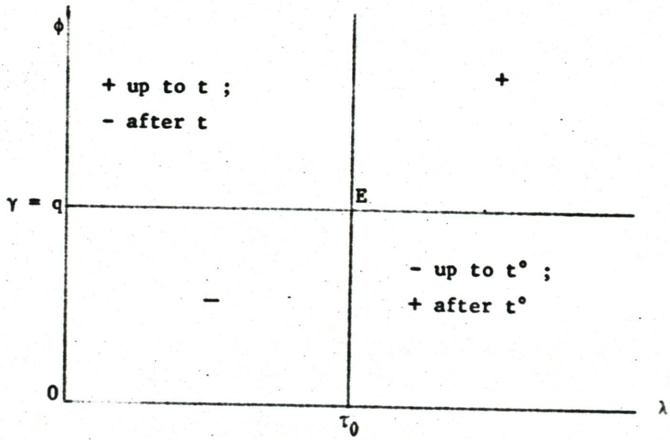

Table 1-3-3 or its alternative Figure 1-3 offers the following interesting observations. Maintenance of external equilibrium in addition to the assumed condition of domestic equilibrium requires not only that the potential policy parameter τ_0 be equal to the given growth rate of exports, but also that other policy variables q and γ be set equal to the given terms of trade ϕ. In other words, any combination of terms of trade and a growth rate of exports represented by a point in one of the four quadrants of Figure 1-3-1 and 1-3-2 fails to cause realization of external as well as domestic equilibrium. Such an ideal combination of terms of trade and a growth rate of exports can be found in Figure 1-3-3 and nowhere else. The point E in Figure 1-3-3 corresponds obviously to the condition (1-3-1). Any deviation of one or both of the two coordinates away from E results in eventual indefinite 'imbalance' of the balance of payments on current account of the

economy with domestic equilibrium.

Comparison of the north-west quadrant with the south-east one in each of the four diagrams leads us to the interesting observation that in the long run a relatively high growth rate of exports $\lambda > \tau_0$ tends to contribute more to the improvement of the balance-of-payments position than a relatively high net terms of trade $\phi > q$. This tendency applies to the marginal change of the balance of payments on current account as well. Implications of this observation for policy are that the growth rate of exports is a more effective measure than the terms of trade in correcting or improving the balance of payments position in the long run. This is true of any of the economies, advanced or underdeveloped, because of the universal character of the outcome.

The left-hand half-space of each of the three diagrams tells us that, when the capacity-increasing effect of the economy is so low that scarcely any induced investment is noticeable, that is to say, $\rho \approx 0$ and $\lambda \ll h/\rho$, the balance of payments position tends to grow continuously worse. One of the reasons is excessive imports-increasing effect of growing effective demand.

Table 1-3-3 or Figure 1-3 would constitute useful reference material in projecting the amount of financial foreign-aid capital needed for an economy on an import-export gap basis[9]. For instance,

[9] As for estimation of capital needs along this approach, see e.g. B. A. Balassa, Trade Prospects for Developing Countries, The Economic Growth Center, Yale Univ., Richard D. Irwin, 1964; B. A. Balassa, 'The Capital Needs of the Developing Countries,' Kyklos, Vol. XVII, 1964; H. Kitamura, 'Trade and Capital Needs of Developing Countries and Foreign Assistance,' Weltwirtschaftliches Archiv, Bd. 97, 1966, Heft 1.

when the economy happens initially to be in the section (c-2-2; a-2), (c-1; a-2) or (b;a-2) in Table 1-3-3, or in the southeast quadrant of Figure 1-3, the observed temporary deficit cannot become a cause of large continuous long-run capital needs.

By way of comparison it is interesting to analyze how the domestic gap in output (income) appears and changes from case to case on the assumption that the condition of external equilibrium growth (1-3-1) is satisfied. The upshot, summarized in Table 1-3-4, tells us the fact that, so long as external equilibrium obtains in an actual economy, the higher the growth rate of exports and the terms of trade are, the greater the inflationary gap will tend to appear, and vice versa. The inflationary gap is used here in the sense that the actual output (income)[10] which is Y_e exceeds that output (income) with domestic equilibrium $G_w = G_n$ which precludes by definition any inflationary or deflationary divergence, implying fully utilized capital and fully employed labor. The deflationary gap in turn connotes underproduction in the actual economy, shortage of income relative to its domestic equilibrium level and hence weaker total spending forces.

[10] Net national output coincides with national income on account of the adopted assumption in Sec. 1 to neglect factor incomes and unilateral transfers.

Table 1-3-4

The Domestic Gap in Output (Income)
when the External Equilibrium is realized continuously

Inf. Gap = Inflationary Gap: $Y_a = Y_f > Y_e$;

Def. Gap = Deflationary Gap: $Y_e > Y_f = Y_a$

Y_a : actual output (income)

<table>
<tr><td colspan="4">c
$\lambda < h/\rho$</td><td rowspan="3">b
$\lambda \geqq \frac{h}{\rho}$</td><td rowspan="3"></td></tr>
<tr><td rowspan="2">c-3
$\lambda < \tau_0$</td><td colspan="2">c-2 $\lambda = \tau_0$</td><td rowspan="2">c-1
$\lambda > \tau_0$</td></tr>
<tr><td>c-2-1
$\gamma \geqq \phi$</td><td>c-2-2
$\gamma < \phi$</td></tr>
<tr><td colspan="2">Inf. Gap up to t° ;
Def. Gap after t°</td><td colspan="3">Inf. Gap</td><td>a-1
$\phi > q$</td></tr>
<tr><td colspan="2">Def. Gap (or no Gap[11])</td><td colspan="3">Def. Gap up to t°
Inf. Gap after t°</td><td>a-2
$\phi \leqq q$</td></tr>
</table>

[11]No Gap occurs only when the condition (1-3-1) is met.

Section 4

Capital Movements with Respect to Domestic and External Equilibrium Growth

Table 1-3-3 and Table 1-3-4 give a clue to the question: how capital movements can help attain one of the equilibria when the other is realized. Let us begin with Table 1-3-3. This shows how much imbalance tends to appear potentially in the balance of payments under different situations. When the growth rate of exports and the terms of trade are sufficiently high, a growing surplus tends to appear in the balance of payments. In such an economy the imbalance can be corrected by capital exports of the current account (aid, donations, gifts and the like). When the capital exports are continuously adjusted to be equal to the export surplus, the result is a realization of external equilibrium growth in addition to the domestic equilibrium growth. On the other hand, when the growth rate of exports and the terms of trade are sufficiently low, the imbalance takes the form of a growing import surplus of the income account. A solution to this external disequilibrium can be given by capital imports. If the capital imports are adjusted to be equal to the import surplus, there will be an external equilibrium as well.[12]

Let us next turn to Table 1-3-4. Both the terms of trade and the

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[12] This is the idea behind the so-called Import-Export Gap analysis of capital needs of developing countries. cf. H. Kitamura, 'Trade and Capital Needs of Developing Countries and Foreign Assistance,' Weltwirtschaftliches Archiv. Bd. 97, 1966, Heft 1; B. A. Balassa, 'The Capital Needs of the Developing Countries,' Kyklos, Vol. XVII, 1964.

growth rate of exports being sufficiently high, domestic capital and labor tend to be more than fully utilized. This leads to potential inflationary pressures. The excess of the potential spending forces (investment and exports) over the potential non-spending forces (saving and imports) can be corrected by importation of foreign saving.[13] On the other hand, if the potential non-spending forces exceed the potential spending forces, the gap can be filled by exportation of domestic saving.

[13] This is a basic idea of the so-called Investment-Saving Gap analysis of capital needs of developing countries.

Appendix to Chapter I

Disaggregation of Macro Equations

This addendum is intended for the clarification of the so far more or less neglected aspect of macro-economic analysis: the process of disaggregation. This methodological field of analysis is particularly relevant to the theory of growth, for, as Harrod in passing puts it, "Growth is the aggregated effect of a great number of individual decisions."[1]

Those aggregative equations which are considered as resultants of individual decisions and actions are in the present system the consumption function (1-1-2), the investment function (1-1-3) and the import function (1-1-5); we are going to show in explicit form the process of disaggregation in each of these three fundamental macro equations.

The theory of aggregation is broadly divided into two approaches: the consistency approach and the analogy approach. The former, represented by L. R. Klein[2], tries to construct those aggregates which are consistent with the given micro theory and the given macro property; the latter, proposed by H. Theil[3], derives in turn the macro relation from the given natural aggregates and the given micro theory. Comparison of the two approaches with each other reveals that the analogy

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[1] R. F. Harrod, Towards a Dynamic Economics, London, 1951, p. 76. Thus Harrod is aware of the problem of aggregation, though Keynes subsumed it in his aggregative analysis.

[2] L. R. Klein, 'Macroeconomics and the Theory of Rational Behavior,' Econometrica, Vol. 14, 1946, pp. 93-108.

[3] H. Theil, Linear Aggregation of Economic Relations, Amsterdam, 1954.

approach proves to be superior in point of applicability, because the aggregates used by H. Theil are the usual ones, such as sums, averages, index numbers and so on. This advantage being taken into account, the analogy approach will be resorted to extensively in the following analysis.

Section 1

Disaggregation of the Consumption Function

Symbols:

C_i: consumption expenditure of the i-th individual

$\overline{C}_i$: autonomous consumption expenditure of the i-th individual

c_i: marginal propensity to consume of the i-th individual

Y_i: income of the i-th individual

$\sum C_i = C$: total consumption (autonomous and induced)

$\sum \overline{C}_i = \overline{C}$: total autonomous consumption

$\sum Y_i = Y$: total income

N : number of individual consumers $(i = 1, \cdots, N)$

The assumption is made that an individual consumer's behavior is in equilibrium explained by two factors. The one is subject to some non-income factors, while the other is influenced totally by the level of his or her income, so that

$$C_i = \overline{C}_i + c_i Y_i + u_{c,i} \; . \qquad (1\text{-}A\text{-}1)$$

The micro random element, $u_{c,i}$, indicates all the factors not

explained by the above two factors and is assumed to satisfy the unbiasedness condition $\sum u_{c,i} = 0$.

For the purpose of connecting the micro consumption relation with the macro one, we introduce, following H. Theil, the auxiliary least-squares equation of the form

$$Y_i = A_i + B_i Y + V_i \, . \tag{1-A-2}$$

Here A_i (regression coefficient), that part of the individual income which is independent of the total income, satisfies the condition: $\sum A_i = 0$, and B_i (regression coefficient) corresponds to what we may call a marginal distribution ratio of income: $\partial Y_i/\partial Y = B_i$, satisfying the condition: $\sum B_i = 1$. The remaining V_i represents the random element satisfying the condition $\sum V_i = 0$.

By dint of the statistically fitted relation (1-A-2) we can easily show how each micro coefficient affects the corresponding macro counterpart. Substituting (1-A-2) for (1-A-1) and adding over all individuals, we can determine the macro parameters of the macro consumption function (1-1-2):

$$\overline{C} = \sum \overline{C}_i + \sum c_i A_i = \sum \overline{C}_i + N \operatorname{cov}(c_i, A_i)$$

$$c = \sum c_i B_i = (1/N) \sum c_i + N \operatorname{cov}(c_i, B_i)$$

$$u_c = 0 = \sum u_{c,i} + \sum c_i V_i = \sum u_{c,i} + N \operatorname{cov}(c_i, V_i). \tag{1-A-3}$$

The equation (1-1-2) has turned out to be based on the implicit assumption that the micro disturbances $u_{c,i}$ of (1-A-1) add up to zero, and the coefficient c_i is uncorrelated with V_i of (1-A-2).

Now it is possible to express the aggregated effect of individual consumer behaviors in the form:

$$C = \sum \overline{C}_i + N \text{ cov } (c_i, A_i) + (1/N) [\sum c_i + N \text{ cov } (c_i, B_i)] Y + u_c \quad (1\text{-A-}4)$$

where $u_c = \sum u_{c,i} + N \text{ cov } (c_i, V_i) = 0$ (assumed).

Expression (1-A-4), though mathematically sufficient to indicate the process of aggregation, does not necessarily lead us to easy economic interpretations. This is where we proceed to inquire into the conditions of perfect aggregation proposed originally by H. Theil.

Instead of his original method, however, we employ here the simpler technique set forth by H. A. J. Green[4]. The condition of perfect aggregation boils down to the statement that there is no contradiction whatever between the two aggregates of consumption; the first of the two aggregates denoted by C^1 is determined by the assumed macro relation (1-1-2) and the definition of the total income $Y = \sum Y_i$, while the other denoted by C^2 is specified by the given micro relation (1-A-1) and the definition of the total consumption $C = \sum C_i$. Since we have

$$C^1 = \overline{C} + cY = \overline{C} + c(\sum Y_i)$$

and

$$C^2 = \sum C_i = \sum (\overline{C}_i + c_i Y_i + u_{c,i}) = \sum \overline{C}_i + \sum c_i Y_i ,$$

[4] H. A. J. Green, Aggregation in Economic Analysis, Princeton Univ. Press, 1964, p.36 (Theorem 7).

the requirement of the perfect aggregation is expressed by the invariable satisfaction of the relation:

$$d\,C^1 \equiv d\,C^2 \quad \therefore \quad c \sum d\,Y_i = \sum c_i\, d\,Y_i = \sum c_j\, d\,Y_j .$$

Since $d\,Y_i$ and $d\,Y_j$ can assume any arbitrary value, the identity is reducible to

$$c = c_i = c_j \quad (i,j = 1,\cdots,N)\,. \tag{1-A-5}$$

That is, the condition of perfect aggregation is satisfied when the individual marginal propensity to consume is equal for all consumers and, at the same time, equal to the macro marginal propensity to consume. It is easy to show that (1-A-5) is also the sufficient condition for perfect aggregation. Once the necessary and sufficient condition (1-A-5) is satisfied, we see our purpose here accomplished; the process of disaggregation of the macro consumption function is easy enough to comprehend.

Section 2

Disaggregation of the Investment Function

Symbols:

- K_i: capital stock of the i-th producer
- ρ_i: micro capital coefficient, marginal = average
- Y_i: output by the i-th producer
- $K = \sum K_i$: total capital stock
- ρ: macro capital coefficient, marginal = average

Since public investment, innovatory investment or other autonomous investments do not lend themselves to easy disaggregation, we may ignore the intercept of the macro investment function (1-1-3), that is, $\bar{I} = 0$. Being integrated with respect to time, (1-1-3) becomes, with the integral constant neglected,

$$K = \rho Y . \tag{1-A-5}$$

The micro investment function is accordingly subsumed to be

$$K_i = \rho_i Y_i , \tag{1-A-6}$$

which is based on the observation that an individual producer adjusts the increase of his capital stock to the current increment of output.

The relationship between a micro output by an individual producer and the macro output aggregated is fitted statistically. The present auxiliary least-squares equation becomes

$$Y_i = Q_i Y + W_i \quad [\sum Q_i = 1, \sum W_i = 0] \tag{1-A-8}$$

where Q_i (regression coefficient) shows roughly the share of the i-th producer's output in the total output, and where W_i (random element) is assumed to be independent of any other coefficient. Substituting Y_i (1-A-8) for Y_i (1-A-6) and aggregating over all producers we reach

$$\sum K_i = K = (\sum \rho_i Q_i) Y + \sum \rho_i W_i ,$$

where $\sum \rho_i W_i = 0$ (assumed). Thus the macro capital coefficient has turned out to be

$$\rho = \sum \rho_i Q_i = (1/N) \sum \rho_i + N \operatorname{cov} (\rho_i, Q_i) .$$

Perfect aggregation requires the following identity to be satisfied for any arbitrary change of the micro output

$$d\,K^1 = \rho \sum d\,Y_i \equiv \sum \rho_i\, d\,Y_i = d\,K^2$$

where

$$K^1 = \rho Y = \rho \sum Y_i \quad \text{and} \quad K^2 = \sum K_i = \sum \rho_i Y_i\,.$$

Therefore the necessary and sufficient condition for perfect aggregation boils down to

$$\rho = \rho_i = \rho_j \quad [i,j = 1,\cdots,N]. \tag{1-A-9}$$

In other words, this condition means that every producer behaves according to the same capital coefficient which is also equal to its macro couterpart. In case (1-A-9) is satisfied, we can easily show that the macro capital stock function (1-A-5) is completely disaggregatable into each micro function.

Section 3

Disaggregation of the Import Function

Symbols:

M_i: imports of an i-th individual

$\overline{M}_i$: autonomous imports of an i-th individual

m_i: marginal propensity to import of an i-th individual

Y_i: income of an i-th individual

$\sum M_i = M$: total imports

$\sum \overline{M}_i = \overline{M}$: total autonomous imports

$\sum Y_i = Y$: total income

The individual import function is of the same nature as the micro consumption function, the only difference being that in the former case payments are made on foreign goods and services. We may well reach directly the assumed micro import function:

$$M_i = \overline{M}_i + m_i Y_i + u_{m,i} . \qquad \text{(1-A-10)}$$

Again we postulate the auxiliary least-squares equation

$$Y_i = A_i + B_i Y + V_i \; [\sum A_i = 0, \sum B_i = 1, \sum V_i = 0] . \qquad \text{(1-A-11)}$$

By dint of (1-A-10) and (1-A-11) it is possible to specify the macro import function

$$M = \overline{M} + mY + u_m$$

where

$$\overline{M} = \sum M_i + \sum m_i A_i = \sum \overline{M}_i + N \text{ cov } (m_i, A_i)$$

$$m = \sum m_i B_i = (1/N) \sum m_i + N \text{ cov } (m_i, B_i)$$

$$u_m = \sum u_{m,i} + \sum m_i V_i = \sum u_{m,i} + N \text{ cov } (m_i, V_i)$$

$$u_m = 0 \text{ (assumed).}$$

The condition of perfect aggregation here will turn out to be

$$m = m_i = m_j \quad (i,j = 1,\cdots,N) .$$

That is, when each individual has the same marginal propensity to import, which is also equal to its macro counterpart, we can consider

the macro import function (1-1-5) as perfectly disaggregatable into each corresponding micro function.

CHAPTER II

CAPITAL IMPORTS AND THE INVESTMENT ALLOCATION OF AN UNDERDEVELOPED ECONOMY

Section 1

Introduction

Table 1-3-3 in Chapter I only implies an aggregate required amount of capital imports as a discrepancy between the theoretical and the actual figures of the balance of payments on current account. The present chapter is, in turn, intended to give a sub-sectoral required amount of capital imports as a difference between the expected and the real figures of the individual investments. This chapter is thus designed for a micro criterion of investment allocation. The analysis proceeds in the context of many sectors which are interdependent on each other in investment activity. Resort is made to what may be called a capital coefficient matrix, or to borrow Prof. Kurihara's coinage, 'Harrod matrix.'[1] This replaces the input-output coefficient matrix of the Leontief open model which is devoid of capacity-increasing elements.[2]

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[1] This is the term invented by him in his effort to multisectorize Harrod's growth model in the light of Leontief's multisectoral methodology. See: Kenneth K. Kurihara, 'The Harrod Matrix and Multisectoral Linear Programming,' Macroeconomics and Programming, London, 1964, Ch. 7. It is possible to retain the Leontief input-output matrix in the following analysis, which, however, makes interpretations less easy.

[2] A disaggregated growth model here will prove to be a major basis of optimizing models of Chapters 3, 5 and 7.

The allocation of investment is of fundamental importance in framing an economic plan for an underdeveloped economy. Various investment criteria have been discussed by several economists. Broadly speaking, these criteria fall into four groups. The first criterion is the turn-over concept which regards choice of investment projects as determined by lowness of capital-output ratio; this theory is represented by J. J. Polak, N. S. Buchanan.[3] The second criterion, the social marginal productivity one, is advocated by A. E. Kahn, H. B. Chenery, who state that investment is to be allocated in such a way that the social marginal productivity of capital is approximately equal in different sectors.[4] The third criterion, represented by W. Galenson, H. Leibenstein, H. Neisser, claims that policy makers should try to equate what they call the marginal per capita reinvestment quotient (net productivity per worker minus consumption per worker)[5]. The fourth view is what is called a time series criterion, developed by A. K. Sen, O. Eckstein, P. A. Samuelson, R. M. Solow. They specify some objective function about gross output or consumption

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[3] J. J. Plak, 'Balance of Payments of Countries Reconstructing with the Help of Foreign Loans,' QJE, Feb. 1953; N. S. Buchanan, International Investment and Domestic Welfare, New York, 1945.

[4] A. E. Kahn, 'Investment Criteria in Development Programmes,' QJE, Feb. 1951: H. B. Chenery, 'The Application of Investment Criteria,' QJE, Feb. 1953.

[5] W. Galenson and H. Leibenstein, 'Investment Criteria, Productivity and Economic Development,' QJE, Auf. 1955; H. Neisser, 'Investment Criteria, Productivity and Economic Development,' QJE, Nov. 1955.

to be optimized within a stipulated planning horizon.[6] By contrast the theory to be developed here is based upon the simpler criterion of domestic equilibrium. The sub-sectoral investments, then, are optimal in the sense that they warrant, on a micro as well as macro basis, a continuous realization of the ex-ante equality of the time series of the spending forces with the time series of the non-spending forces. Although it is not a result of optimization of some objective function, it helps to obviate those restrictive assumptions otherwise needed. The Harrod-Domar model like Keynes' own model has as one of its limitations an overaggregative nature. It fails to single out inefficient sectors which militate against overall or individual target. It remains of little operational significance unless generalized to include a number of different sectors. As mentioned before, Harrod points out that "economic growth is the aggregated effect of a great number of individual decisions."

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[6] A. K. Sen, Choice of Technique, An Aspect of the Theory of Planned Economic Development, Blackwell, London, 1960; O. Eckstein, 'Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economics,' QJE, Feb. 1957; P. A. Samuelson and R. M. Solow, 'A Complete Capital Model Involving Heterogeneous Capital Goods,' QJE, Nov. 1956.

Section 2

The Model

Notations:

$E = [1]$, the unit row vector of an appropriate order

$C = \{C_i\}$, the column vector of consumption activity

$\bar{c} = \{\bar{c}_i\}$, the column vector of autonomous consumption activity

$c = [c_i]$, the diagonal matrix of the marginal propensities to consume

$Y = \{Y_i\}$, the column vector of production activity

$I = \{I_i\}$, the column vector of total investment activity

$J = [\rho_{ij}]$, the matrix of the fixed intersectoral capital coefficient, the amount of the i-th capital input required per unit production of the j-th output in investment activity, with the properties

$$\rho_{ij} \geqq 0 \quad \text{and} \quad \sum_i \rho_{ij} < 1 .$$

$\bar{I} = \{\bar{I}_i\}$, the column vector of autonomous investment activity

$X = \{X_i\}$, the column vector of export activity

$\dot{Y} = \{\dot{Y}_i\}$, the column vector of the marginal change of production activity

$M = \{M_i\}$, the column vector of import activity

$\bar{m} = \{\bar{m}_i\}$, the column vector of autonomous import activity

$m = [m_i]$, the diagonal matrix of the marginal propensity to import

$\epsilon = [\epsilon_i]$, the diagonal matrix of the marginal propensity to export

$\bar{X} = \{\bar{X}_i\}$, the column vector of autonomous export activity $\bar{X}_i = \bar{X}_i(0)e^{\lambda_i t}$

The required dynamic condition for the continuous equalization of the intended investment plus exports and the saving plus imports in the open economy can be written as

$$E\,Y - E\,C + E\,M = E\,I + E\,X\,,$$

or

$$E\,Y = E\,\bar{c} + E\,c\,Y + E\,J\,\dot{Y} + E\,\bar{X} - E\,\bar{m} - E\,m\,Y + E\,I + E\epsilon\,Y \quad (2\text{-}2\text{-}1)$$

For the purposes of analysis, the present study will now discuss two of the ways to show how sub-sectoral investments are determined. The first method begins by specifying the conditions under which perfect aggregation is possible. The Theil-Green conditions of Perfect Aggregation dealt with in the Appendix to Chapter I imply here:

i) the marginal propensity to consume of every individual is equal to each other and also to the macro counterpart, i.e. $c_i = c_j = c$.

ii) the marginal propensity to import of every individual is equal to each other and also equal to the macro counterpart, i.e., $m_i = m_j = m$.

iii) the micro capital-output ratio is equal to each other and also to the macro counterpart, i.e. $\rho_i = \rho_j = \rho$ where $\rho_i = \sum_k \rho_{ik}$ and $\rho_j = \sum_q \rho_{jq}$.

When these three restrictive assumptions are fulfilled, the macro equation (2-2-1) is completely disaggregatable into a sum of

$$Y_i = \bar{c}_i + c_iY_i + \rho_i\dot{Y}_i + \bar{I}_i + \bar{X}(0)_i e^{\lambda_i t} + \epsilon_iY_i - \bar{m}_i - m_iY_i\,.$$

If we further assume the equality of growth rate of exports in each sector, the warranted growth path of the i-th sector can be determined by

$$\text{(i)}\quad Y_i = \frac{\bar{X}(0)_i}{h_i - \lambda\rho_i}e^{\lambda t} + \left[Y_i(0) - \frac{\bar{X}(0)_i}{h} + R_{0,i}\right]e^{(h_i/\rho_i)t} \quad \text{if } \lambda = \lambda_i \neq h_i/\rho_i$$

$$\text{(ii)}\quad Y_i = -\frac{\bar{X}(0)_i}{\rho_i} t\, e^{\lambda t} + \left[Y_i(0) + R_{0,i}\right]e^{\lambda t} - R_0 \quad \text{if } \lambda = \lambda_i = h_i/\rho_i$$

Accordingly the warranted level of the i-th sector investment is to be specified as

$$\text{(i)}\quad I_i = \left[\frac{\bar{X}(0)_i h_i}{h_i - \lambda\rho_i} - \bar{X}(0)_i\right] e^{\lambda t} + h_i\left[Y_i(0) - \frac{\bar{X}(0)_i}{h_i - \lambda\rho_i} + R_{0,i}\right]e^{(h_i/\rho_i)t}$$

$$\text{(ii)}\quad I_i = \left[h_i Y_i(0) + h_i R_{0,i} - \frac{\bar{X}(0)_i h_i}{\rho_i} - \bar{X}(0)_i\right]e^{\lambda_i t}$$

In the light of these warranted investment paths we can determine the relative deficiency or excess of each sectoral current investment.

It is possible to judge the effect of increasing capital productivity on the growth rate of the i-th sector. This increase in capital productivity can be regarded as the accompaniment of an absorbed foreign investment activity as discussed in Case 4 of the Appendix to Chapter II.

(i) If $h_i Y_i > X_i - h_i R_{0,i}$, $\frac{\partial G_i}{\partial \rho} > 0$.

That is, in case the growth rate of the equilibrium output is large enough, relative to that of exports, the increase in the marginal capital productivity tends to result in the increase of the growth rate of the i-th output.

(ii) If $h_i Y_i < X_i - h_i R_{0,i}$, $\frac{\partial G_i}{\partial \rho} < 0$.

To wit, when the growth rate of exports exceeds that of output sufficiently, the increase in the marginal capital productivity tends to decrease the growth rate of output of the i-th sector. This is again, as is the case in Table 1-1-2, a reflection of the assumption that all indirect unquantifiable effects of export expansion through some sort of structural adjustment are neglected.

So much for the first method. The second method proceeds independently of the theory of aggregation upon which the first one is based. We take it for granted that sectoral investment activities are mutually dependent on each other. We begin by assuming the matrix-form relationship

$$Y = \bar{c} + c\,Y + J\,\dot{Y} + \bar{I} + \bar{X} + \epsilon\,Y - \bar{m} - m\,Y . \qquad (2\text{-}2\text{-}2)$$

This is distinguished from the scalar expression (2-2-1). In order to solve this simultaneous first-order differential equation let (2-2-2) be rewritten as

$$J\,\dot{Y} - H\,Y = d , \qquad (2\text{-}2\text{-}3)^{7}$$

[7]When capital imports enter the model at the outset, the required relationship becomes $J\,\dot{Y} - H\,Y = d$, with d defined as $\{-\bar{X} + \bar{m} - \bar{c} - \bar{I} - K_f\}$, where K_f is the column vector of capital im ports.

where

$$H = -\begin{bmatrix} h_1 & - & - & 0 \\ \vdots & \ddots & & \vdots \\ 0 & - & - & h_n \end{bmatrix}, \quad h_i = 1 - c_i + m_i - \epsilon_i \quad (i = 1,\cdots,n),$$

$$\text{and} \quad d = \begin{bmatrix} -\bar{X}_1 + \bar{m}_1 - \bar{c}_1 - \bar{I}_1 \\ \vdots \\ -\bar{X}_n + \bar{m}_n - \bar{c}_n - \bar{I}_n \end{bmatrix}.$$

The matrix H is assumed to be nonsingular. This assumption is plausible, because h_i is the sum of the marginal propensities to save and to import minus the usually smaller marginal propensity to export pertaining to the i-th sector.

To find the particular integrals let us try constant solutions of (2-2-3) $\bar{Y} = \{\bar{Y}_i\}$, which implies $\bar{Y} = \{\dot{\bar{Y}}_i\} = 0$. Since H is non-singular,

$$\bar{Y}, = -H^{-1} d . \qquad (2\text{-}2\text{-}4)$$

Next let us look for complementary functions. The trial solutions $Y_i = f_i e^{rt}$ $(i = 1,\cdots,n)$, which imply $\dot{Y}_i = r\, f_i e^{rt}$, are plugged into the homogeneous version of (2-2-3)

$$J\,\dot{Y} - H\,Y = 0,$$

to obtain

$$[J\,r - H]\left\{f_i\, e^{rt}\right\} = 0\,,$$

where f_i is an arbitrary constant $(i = 1,\cdots,n)$. As our objective

is to get nontrivial solutions of f_i's , it is necessary that the determinant vanishes, i.e.

$$|J r - H| = 0 .$$

This n-th order polynominal characteristic equation gives a column vector of the characteristic roots $r_j (j = 1, \cdots, n)$. The condition for each characteristic root to be positive is specified by the Routh Theorem, which requires a matrix formed by the coefficients of the above polynominal to follow a certain rule[8].

The general solution of (2-2-3) then takes the form

$$Y = F Q + \overline{Y} , \tag{2-2-5}$$

where $Y = \{Y_i\}$, $F = [f_{ij}]$ (the matrix of the coefficients specified by initial conditions) and $\overline{Y} = \{\overline{Y}_i\}$. The corresponding warranted investment path is specified as

$$I = J \dot{Y} = H Y + d , \tag{2-2-6}$$

where Y and $\dot{Y}$ are given by (2-2-5).[9]

J. Tinbergen and H. C. Bos in their book, Mathematical Models of Economic Growth (McGrawhill, 1962), stress the importance of the lags

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[8] P. A. Samuelson, Foundations of Economic Analysis, Harvard Univ. Press, 1947, pp. 229-235.

[9] The general solutions of sectoral outputs and investments thus derived can be used for predictive and planning purposes. A detailed work by Leif Johansen includes the Solution Matrix of the generalized input-output system of Norway: A Multi-sectoral Study of Economic Growth, North-Holland, Amsterdam, 1964, p. 74, p. 170. The problem of stability of the equilibrium solutions (2-2-5) and (2-2-6) is something that may be added. See: Dale W. Jorgenson, 'Growth and Fluctuations: A Causal Interpretation,' QJE, 1960, p. 421.

in investment (p. 12). Consideration of the gestation lag is, as they assert, essential in planning of investment. Of practical relevance, further, is the diversity of the periods of gestation in different sectors. The investment process in one sector may take a much longer time than that in another sector. We now investigate the consequences of the existence of different investment lags on the equilibrium sectoral outputs and investments.

The investment function of the i-th sector needs now to be transformed into

$$\dot{I}_i = -g_i \left(I_i - \sum_j \rho_{ij} Y_j \right)$$

where $g_i > 0$ implies the speed of response of the i-th sector. In proportion to the lag of actual investment I_i behind the potential investment $\sum \rho_{ij} \dot{Y}_j$, the actual investment shows its marginal change, the ratio of proportionality being g_i. Let the diagonal matrix of the speed of response be

$$G = \begin{bmatrix} g_1 & & 0 \\ & \ddots & \\ 0 & & g_n \end{bmatrix} .$$

Then the characteristic equation becomes

$$|G\,J\,r - H| = 0 .$$

The sectoral equilibrium output now is

$$Y = P\,S + \overline{Y}, \qquad (2\text{-}2\text{-}7)$$

where $\bar{Y}$ is the column vector of the particular solutions of the new differential system, P the $n \times n$ matrix of the coefficients specified by initial conditions and $S = \{e^{s_j t}\}$, s_j being the j-th characteristic root of the new equation system. Thus the equilibrium sectoral investment turns to

$$I = [G J - H]\{\dot{Y}\} = G H Y + z , \qquad (2\text{-}3\text{-}8)$$

where z is the column vector of exogenous elements corresponding to d .

To exemplify the above general solutions let us proceed to postulate a simple two-sector model. The first sector is assumed to be characterized by a higher marginal capital productivity as a consequence of the absorbtion of foreign advanced productive factors. The second sector, unaffected by such opportunities, remains behind in the capital productivity. To sharpen the comparison, the two sectors are assumed to be possessed of the same non-spending ratio, i.e. $h_1 = h_2$. Let the capital coefficient matrix be

$$J = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 2.5 & 3 \end{bmatrix}$$

and the marginal net non-spending ratio matrix be

$$H = -\begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix} = -\begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$$

Then the characteristic roots are given by

$$r_1 = 1.544$$
$$r_2 = 0.039$$

with the sectoral outputs

$$Y_1 = f_{11}e^{1.544t} + f_{12}e^{0.039t} + \overline{Y}_1$$
$$Y_2 = f_{21}e^{1.544t} + f_{22}e^{0.039t} + \overline{Y}_2$$

and with the sectoral investments

$$I_1 = 2\,\dot{Y}_1 + 5\,\dot{Y}_2 = (2.088f_{11} + 7.720f_{21})\,e^{1.544t} + 2\,\dot{\overline{Y}}_1 + (0.078f_{12} + 0.195f_{22})\,e^{0.039t} + 5\,\dot{\overline{Y}}_2$$

$$I_2 = 2.5\,\dot{Y}_1 + 3\,\dot{Y}_2 = (3.860f_{11} + 4.632f_{21})\,e^{1.544t} + 2.5\,\dot{\overline{Y}}_1 + (0.097f_{12} + 0.117f_{22})\,e^{0.039t} + 3\,\dot{\overline{Y}}_2\,.$$

These exemplified solutions show in concrete form the process of joint determination of sectoral outputs and investments.

The addition of the gestation lags makes the two-sector model more meaningful. The first sector, which is further assumed to be a strategic for rapid development, is marked by a much smaller speed of response. The hydroelectric sector, for instance, requires a much

longer gestation period and more lumpy investment than a non-strategic sector such as a commercial one, implying that the actual deficit between the actual rate of investment and the potential rate of investment, i.e. $-[I_i - \sum \rho_{ij} Y_j]$, causes a smaller change in the rate of investment of the first strategic sector than in the non-strategic second sector. The adjustment of change in inventory investment in response to the above deficit can proceed at a more rapid rate than the adjustment of change in construction of fixed capital such as a dam. Added to the above data is thus the speed-of-response matrix

$$G = \begin{bmatrix} 1/4 & 0 \\ 0 & 2 \end{bmatrix}.$$

The characteristic roots now change to

$$r_1 = 0.06 + 0.162\, i$$

$$r_2 = 0.06 - 0.162\, i$$

with the sectoral outputs

$$Y_1 = (p_{11} \cos 0.162t + p_{12} \sin 0.162t) e^{0.06t} + \bar{Y}_1$$

$$Y_2 = (p_{21} \cos 0.162t + p_{22} \sin 0.162t) e^{0.06t} + \bar{Y}_2$$

and with the sectoral investments

$$I_1 = -0.1\, A_1 [0.06\, e^{0.06t} \cos(0.162t + \psi_1) - 0.162\, e^{0.06t} \sin(0.162t + \psi_1)] +$$
$$1.25\, A_2 [0.06\, e^{0.06t} \cos(0.162t + \psi_2) - 0.162\, e^{0.06t} \sin(0.162t + \psi_2)]$$

$$I_2 = 5\,A_1[0.06\ e^{0.06t}\cos(0.162t+\psi_1) - 0.162\ e^{0.06t}\sin(0.162t+\psi_1)] +$$

$$5.4A_2[0.06\ e^{0.06t}\cos(0.162t+\psi_2) - 0.162\ e^{0.06t}\sin(0.162t+\psi_2)]\ ,$$

where p_{ij} $(i,j = 1,2)$ is determined by initial conditions,

$p_{11} = A_1\cos\psi_1$, $p_{12} = -A_1\sin\psi_1$, $p_{21} = A_2\cos\psi_2$ and

$p_{22} = -A_2\sin\psi_2$. In these examples it turns out that the sectoral outputs and investments with the investment lags introduced tend to show explosive oscillaroty movements over time, whereas both tend to increase monotonically at increasing rates before the introduction of the investment lags.

A simple criterion of investment allocation has thus been brought to light. An analytical method has been explicated to meet the objective of the present chapter, which is, to repeat, to determine each sectoral investment in such a way that the spending side and the non-spending side of each sector are equal to each other. Among the data needed in predicting the desired sectoral investment, the capital coefficient matrix would be most difficult to come by. Once this 'Harrod matrix' along with the other sectoral parameters is specified, it would be a matter of technical computation to estimate the terminal amount of each sectoral investment and hence that of total investment required over the time horizon of a multi-year plan. Based on this prediction the planning agency can determine how much real investment attributable to advanced foreign productive factors is to be made in each sector, not to mention how much financial capital import is to be

made sectorally in advance. In a word, the warranted subsectoral investment (2-2-6) or (2-2-8) constitutes the gist of the present section.

The investment criterion based on the optimum growth theory seems to the author to be more appropriate for an affluent advanced economy which finds itself in Rostow's 'high mass consumption' stage. For its objective function is usually the flow of consumption, the utility of consumption, the integral of discounted or undiscounted utility of consumption, and so on. The underdeveloped economy is typically characterized by scarce real as well as financial capital; for such an economy the optimum growth theory lacks its validity, a fortiori when compared with the Harrod-Domar theory.

Appendix to Chapter II

How to Use Foreign-Aid Capital

The analysis in this appendix attempts to bring out various effects of different uses of a given amount of foreign aid-capital. One of the assumptions of Chapter I, i.e. neglect of unilateral transfers, is now relaxed. An underdeveloped economy aims typically at rapid growth, with far less importance attached to stability problems more or less common to advanced economies. We will show the utilization of the unilateral foreign aid-capital which gives rise to a maximum rate of growth of output among several simplified situations.[1]

In the underdeveloped economy, commodities are needed for consumption, production, or investment, but at the same time, it may suffer from a scarcity of foreign exchange reserves. Alternative uses of the assistance may be represented by the following four cases:

Case 1: Addition to foreign exchange reserves, considered as increase of national income.

Case 2: Importation of foreign consumer goods (including foodstuffs, durable and non-durable goods, and final consumer goods), considered as giving no net effect on national income or output.

Case 3: Importation of raw materials and intermediate goods for productive purposes, considered as increase of net national output.

Case 4: Importation of capital goods for investment purposes, considered as influencing the domestic investment function.

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[1] A criterion of aid usage involving political factors is proposed by Charles Wolf. His objective function for economic aid is based on the assumption that the United States is interested in maximizing the "political" returns it can gain from aid. Foreign Aid: Theory and Practice in Southern Asia, Princeton, N. J. 1960, Ch. 8.

Tha analysis is based on the model discussed in the preceding chapter -- the Neo-classical assumption of substitutability of resources in response to price change remains neglected and added foreign consumer goods, for instance, offers no inducement to inter-sectoral resource shifting in an economy in which the market mechanism is still underdeveloped (cf. Case 2).

The notations being the same as those in Chapter 1, all that is required is to designate the real value of the foreign aid-capital as A and its money value as $p\,A$ where p is some price index relevant to A. The structure of the underdeveloped economy now consists of (1-1-1), (1-1-2), (1-1-3), (1-1-4), (1-1-5) and the variant of (1-1-6):

$$B = p_x X - p_m M + p\,A\,.$$

The solution of output derived in (1-1-8) shall be utilized, where the suffix e attached to Y_e may now be omitted so long as no misunderstanding occurs thereby. The relationship between the growth rate of exports and that of domestic equilibrium product, Table 1-1-2, will prove to play an important role in the subsequent investigation.

Case 1: Addition to Foreign Exchange Reserves

The government of the developing economy receives an autonomous and anonymous financial assistance and exchanges it with the central bank to obtain its own currency. The velocity of circulation of money and the price level being held constant, it can be treated as an

increase in the total real earnings of the nationals of the economy by the amount of the financial assistance. Y in this case denotes domestically earned income and $Y + A$ national income. This national income will influence the consuming and the importing behaviors, so that the consumption and the import functions take now the respective forms

$$C = \bar{c} + c(Y + A)$$

and

$$M = \bar{m} + m(Y + A) .$$

This would correspond to such a closed economy as the one in which a net increase in autonomous negative tax (transfer payments) affects people's general consuming behavior directly. The equilibrium solution of output here is derived by solving the differential equation

$$\rho \dot{Y} - h Y = -X + h R_1$$

where

$$R_1 = \frac{\bar{m} - \bar{c} + (m - c)A - \bar{I}}{h}$$

Case 2: Importation of Foreign Consumer Goods

The unilateral foreign capital imported, which is spent domestically in Case 1, is here assumed to be spent immediately on importing consumer goods from abroad. This would be similar to the hypothetical case of a closed economy in which an initial increase in autonomous

government spending is immediately counteracted by an autonomous increase in tax revenue. Hence the equilibrium level of Y is now derived by solving

$$\rho \dot{Y} - h Y = -X + h R_2$$

where

$$R_2 = R_0 = \frac{\bar{m} - \bar{c} - \bar{I}}{h}$$

Case 3: <u>Importation of Raw Materials and Intermediate Goods for Productive Purposes</u>

The raw materials and the intermediate goods can be regarded as a source of indirect production. This is designated by H. G. Johnson as 'import content'[2]. Y in this case denotes net national product and Y - A domestically earned income or net domestic product. It is assumed that in this case the net domestic product influences the production of consumer goods, domestic or foreign. Therefore

$$C = \bar{c} + c(Y - A)$$

and

$$M = \bar{m} + m(Y - A) .$$

The equation to be solved becomes now

$$\rho \dot{Y} - h Y = -X + h R_3$$

[2] ibid. p. 125.

where

$$R_3 = \frac{\bar{m} - \bar{c} - (m-c)A - \bar{I}}{h} .$$

Case 4: Importation of Capital Goods for Investment Purposes[3]

The final case coincides with that of a productive direct foreign investment. In order to explicate an absorbed high technological level, we introduce a new investment function of the type

$$\dot{Y} = \frac{1}{\rho_d}(I_d)_d + \frac{1}{\rho_f}(I_d)_f = \left[\frac{1}{\rho_d} + \frac{1}{\rho_f}\frac{(I_d)_f}{(I_d)_d}\right](I_d)_d ,$$

where $(I_d)_d$ is the part of total investment attributable to domestic productive factors, $(I_d)_f$ that part attributable to foreign productive factors, $(I_d)_f$ equal to A, $\frac{1}{\rho_d}$ the average productivity of the former investment, and $\frac{1}{\rho_f}$ the one of the latter investment. The homogeneity of both capitals is subsumed. The equilibrium level of output is then derived as a solution of the equation.

$$\rho\,\dot{Y} - h\,Y = -X + h\,R_4 ,$$

where $\rho = \rho_d$ and $R_4 = \dfrac{\bar{m} - \bar{c} + A\,\rho\left[\frac{1}{\rho_f} - \frac{1}{\rho}\right] - \bar{I}}{h}$. It is interesting to note that

[3]The favorable assumption is made that capital goods imported from advanced economics have a higher "sigma effect" than domestically produced goods.

$$\frac{1}{\rho_f}\frac{(I_d)_f}{(I_d)_d}$$

corresponds to a sort of indicator of technological advance as a result of the absorbed foreign technology. The given domestic investment $(I_d)_d$ produces now more than otherwise.

Comparison of the Four Alternative Cases

Indeed, the conditions giving rise to each of the four results differ from case to case, but a few generalizations are still possible if one bears in mind the mental reservation that the following statements are not absolute, but relative.[4] A similar reservation is usually made when long-run equilibria of perfect competition, monopolistic competition and pure monopoly are compared with each other.

The equilibrium level of output for each case is of the identical form except for the terms R's. To facilitate our understanding we may make a pertinent assumption that the foreign developed economy is at least twice as large in average capital productivity as our underdeveloped economy. This assumption assures the relationship

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[4]Strictly speaking, Y in Case 1 signifies domestically earned income and Y in all the other cases net national product = national income. To make a more meaningful comparison the reader may suggest that the author eliminate Case 1. Even then the general conclusions remain unchanged: it is in Case 4 that the maximum growth rate is attained.

$$\rho\left[\frac{1}{\rho_f} - \frac{1}{\rho}\right] \gtreqless 1 > c - m^{5},$$

which is sufficient to determine the relation between R_4 and $R_{3(1)}$. Once the important relation between the marginal propensity to consume and to import is specified, a quantitative comparison among the four cases becomes possible in the level and the growth rate of output and in the level and the marginal change of the current account, as represented in Table 2-1-1.

Table 2-1-1 leads us to the following observations which include a provisional answer to the question: how to utilize the foreign aid-capital in order to achieve rapid growth (item 2).

1) The highest level of Y is attained in Case 4, i.e. when the aid is used for importation of capital goods. This applies to the level of per capita output, too, on the assumption that the growth rate of population is common to each case.

2) The growth rate of Y becomes highest also when the aid is used for importation of capital goods for investment purposes. This holds for the growth rate of per capita output as well, on the same condition as above. This fourth utilization becomes therefore most desirable for the developing country which can more or less neglect balance-of-payments considerations.

3) The balance of payments on current account tends to be most

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[5] For example, assume $\rho_f = 2$ and $\rho = 5$, and we find the relation $\rho\left(\frac{1}{\rho_f} - \frac{1}{\rho}\right) = 1.5 > 1$.

favorable either when the aid is retained as an addition to foreign exchange reserves, the marginal propensity to import being less than the marginal propensity to consume, or when it is used for importation of raw materials and intermediate goods for productive purposes with the marginal propensity to import exceeding the marginal propensity to consume, but most unfavorable in Case 4.

Table 2-A-1

Comparative Trend Values and Growth Rates of Y (Output) and Comparative Trend Values and Marginal Changes of the Balance on Current Account in the Four Alternative Cases

$\tau_i = \frac{1}{\rho}\left[h - \frac{X(0)}{Y(0) - R_i}\right]$, $G_i = \frac{\dot{Y}_i}{Y_i}$ $(i = 1, 2, 3, 4)$, ϕ = net terms of trade. $t^{\Delta} \geq 0$ and $t^{\circ} \geq 0$ denote respectively the longest of the t_i's to pass until $G_i(t_i) = 0$ and $\dot{B}_i(t_i) = 0$. $t^* \geq 0$ shows the longest of the t_i's to pass until $X(t_i)(\partial\phi/\partial Y) - m = 0$ (see Appendix). The intervals till t^{Δ}, t° and t^*, which preciseness requires to refer to, are to be neglected from the long-run point of development. The marginal propensity to export is assumed to be zero for the sake of argument.

A-5	A-4	A-3	A-2	A-1	
$\lambda < \tau_1$ (I) $\lambda < \tau_3$ (II)	$\tau_1 < \lambda < \tau_2$ (I) $\tau_3 < \lambda < \tau_2$ (II)	$\tau_2 < \lambda < \tau_3$ (I) $\tau_2 < \lambda < \tau_1$ (II)	$\tau_3 < \lambda < \tau_4$ (I) $\tau_1 < \lambda < \tau_4$ (II)	$\tau_4 < \lambda$	
$R_4 > R_3 > R_2 > R_1$, $Y_4 > Y_3 > Y_2 > Y_1$, $B_1 > B_2 > B_3 > B_4$					I
$G_4 > G_3 > G_2 > G_1 > 0$ after t^Δ	$G_4 > G_3 > G_2 > 0 > G_1$ after t^Δ	$G_4 > G_3 > 0 > G_2 > G_1$ after t^Δ	$G_4 > 0 > G_3 > G_2 > G_1$ after t^Δ	$0 > G_4 > G_3 > G_2 > G_1$ after t^Δ	$c > m$
$0 > \dot{B}_1 > \dot{B}_2 > \dot{B}_3 > \dot{B}_4$ after t°	$\dot{B}_1 > 0 > \dot{B}_2 > \dot{B}_3 > \dot{B}_4$ after t°	$\dot{B}_1 > \dot{B}_2 > 0 > \dot{B}_3 > \dot{B}_4$ after t°	$\dot{B}_1 > \dot{B}_2 > \dot{B}_3 > 0 > \dot{B}_4$ after t°	$\dot{B}_1 > \dot{B}_2 > \dot{B}_3 > \dot{B}_4 > 0$ after t°	
$\dot{B}_4 > \dot{B}_3 > \dot{B}_2 > \dot{B}_1 > 0$ after t^* if $\partial\phi/\partial Y_i > 0$	$\dot{B}_4 > \dot{B}_3 > \dot{B}_2 > \dot{B}_1$ after t^* if $\partial\phi/\partial Y_i > 0$				
$R_4 > R_1 > R_2 > R_3$, $Y_4 > Y_1 > Y_2 > Y_3$, $B_3 > B_2 > B_1 > B_4$					II
$G_4 > G_1 > G_2 > G_3 > 0$ after t^Δ	$G_4 > G_1 > G_2 > 0 > G_3$ after t^Δ	$G_4 > G_1 > 0 > G_2 > G_3$ after t^Δ	$G_4 > 0 > G_1 > G_2 > G_3$ after t^Δ	$0 > G_4 > G_1 > G_2 > G_3$ after t^Δ	$c < m$
$0 > \dot{B}_3 > \dot{B}_2 > \dot{B}_1 > \dot{B}_4$ after t°	$\dot{B}_3 > 0 > \dot{B}_2 > \dot{B}_1 > \dot{B}_4$ after t°	$\dot{B}_3 > \dot{B}_2 > 0 > \dot{B}_1 > \dot{B}_4$ after t°	$\dot{B}_3 > \dot{B}_2 > \dot{B}_1 > 0 > \dot{B}_4$ after t°	$\dot{B}_3 > \dot{B}_2 > \dot{B}_1 > \dot{B}_4 > 0$ after t°	
$\dot{B}_4 > \dot{B}_1 > \dot{B}_2 > \dot{B}_3 > 0$ after t^* if $\partial\phi/\partial Y_i > 0$	$\dot{B}_4 > \dot{B}_1 > \dot{B}_2 > \dot{B}_3$ after t^* if $\partial\phi/\partial Y_i > 0$				

4) The observations so far are based on the assumption of constant prices common to the Keynesian approach. When the increasing output accompanies a positive change in the terms of trade, the marginal change of the current account tends in the long run to be most favorable in Case 4 irrespective of the level of the growth rate of exports. Expecially when the growth rate of exports is depressed enough (A-5), the analysis shows that an eventual improvement of the current account is largest in Case 4 (see Appendix (*)).

5) In the final analysis the policy to attain maximum per capita as well as total output without causing balance-of-payments difficulties has proved to be most successfully realized, where the aid is used for importation of capital goods for investment purposes which have higher marginal productivity than the domestically produced capital and some effective measure of self-help is jointly resorted to so as to make and keep the marginal change of the terms of trade positive.

Appendix (*)

For the sake of simplicity let us assume $p_m = 1$, so that the net terms of trade are equal to p_x. The definition of the balance on current account reduces $B_i = \phi\, X(0)\, e^{\lambda t} - (\bar{m} + m\, Y_i) + p\, A$, differentiating which with respect to time we get

$$\dot{B}_i = \left[X(0)\, e^{\lambda t} \frac{\partial \phi}{\partial Y_i} - m \right] \dot{Y}_i + \phi\, X(0)\, \lambda\, e^{\lambda t} \qquad (i = 1, 2, 3, 4).$$

$\frac{\partial\phi}{\partial Y_i}$ indicates the marginal change in the terms of trade attributable to the growth of output. If this is positive, the coefficient of $\dot{Y}_i$ in square bracket turns positive after t^{**}_i, where $X(0)\, e^{\lambda t^{**}_i}\, \partial\phi/\partial Y_i = m_{.}t^{**}$ in Table 2-1-1 is the largest of the t^{**}_i's. A reference to the results of all four cases reveals that in accordance with $Y_4 > Y_{3(1)} > Y_2 > Y_{1(3)}$ the corresponding relation $\dot{B}_4 > \dot{B}_{3(1)} > \dot{B}_2 > \dot{B}_{1(3)}$ tends to be established in the long run. Especially when $\dot{Y}_i$ is each positive (A-5), we find the interesting relation $\dot{B}_4 > \dot{B}_{3(1)} > \dot{B}_2 > \dot{B}_{1(3)} > 0$ established in the long run.

CHAPTER III

CAPITAL IMPORTS AND THE RAPID GROWTH OF AN UNDERDEVELOPED ECONOMY: OPTIMIZING MODELS

Section 1

Introductory Remarks

The analysis so far has left something to be desired in that no target function to be explicitly optimized has yet been introduced. A dynamic input-output model alone is hardly adequate for finding the optimal path of an objective function, as has been pointed out by Prof. Ichimura[1]. The present chapter is designed for presenting those optimal programming models which are relevant to a typical underdeveloped economy. They are expected to make some significant contrast with their counterpart for a typical advanced economy (Ch. 5).

The problems confronting the typical underdeveloped economy are one-sided in the sense that absolute scarcity of capital stock or productive capacity is the major bottleneck of its rapid development, whereas for a typical advanced economy problems are rather two-sided, requiring a dynamic balancing of the demand side and the supply side, as will be seen in Chapters 4 and 5.

Since in this chapter our main concern lies in the rapid growth of the typical underdeveloped economy, the problem of inflation is ignored;

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[1] ECAFE, Programming Techniques for Economic Development, Bangkok, 1960, p. 104.

this approach is justifiable in view of the truth that "there is no convincing historical tendency of any clear association, positive or negative, between the rate of inflation and the rate of economic growth."[2]

Optimizing models of rapid growth can be attacked from two directions. When the target maximum possible growth rate is predetermined exogenously, say, by policy makers, this can enter programming as one of the constraints to be satisfied. When the growth rate can endogenously be determined within a system, this enters the system as the objective function per se. In all of the following programs it is assumed that every objective function is concave, every constraint function g^i is convex and the feasible set $\{x | g^i(x) \leq 0,\ i = 1,\cdots,n;\ x \geq 0\}$ is bounded and nonempty. These three plausible assumptions guarantee the existence of a unique optimal solution set which is a saddle point in the implied Lagrangean function. Since the first two programs below are linear in their objective functions, their dual variables reduce to the corresponding Lagrangean multipliers. Common to all of the programs is the over-time non-linear (exponential) behavior of the sectoral exports, which can take numerical value either at a point of time or in the case of zero growth rates. When they can take such values, the first two programs may justify the term linear programming, inasmuch as the constraints as well as the objective

[2] H. G. Johnson, 'Is Inflation the Inevitable Price of Rapid Development or a Retarding Factor in Economic Growth?' Essays in Monetary Economics, Ch. IX, 1967, p. 282.

functions are linear in independent variables. The last program, however, still remains a non-linear one, for the objective function itself is a non-linear function of outputs.

As for the capital imports, attention is paid on their effects upon the programs of rapid growth. The treatment of the capital imports will prove to exert parametric functions in each of the three following programs optimizing national product, capital accumulation and growth rate per se.

Section 2

Linear Programming for Optimal National Product

One of the typical targets of an underdeveloped economy is to attain a conditional maximization of its net national product or output. The net national product is defined by dint of the notations of Chapter II as follows:

$$E\,Y = E\,\bar{c} + E\,c\,Y + E\,J\,\dot{Y} + E\,X - E\,\bar{m} - E\,m\,Y - E\,K_f. \quad (3\text{-}2\text{-}1)$$

This identity is of course conceptually different from the requirement (2-2-1). Notice that the national income here is $E\,Y + E\,K_f$, exceeding the net national product by the amount of the unilateral capital imports $\sum K_f$. Now that the autonomous consumption $E\,\bar{c}$, the autonomous imports $E\,\bar{m}$, the exogenous exports $E\,\bar{X}$ and the unilateral capital imports $E\,K_f$ are given independently of the output $E\,Y$, the maximization of $E\,Y$ is tantamount to that of

$$E(c-m-\epsilon)Y + E\,J\,\dot{Y} = E\,b\,Y + E\,J\,\dot{Y}$$

where

$$b = c - m - \epsilon .$$

First let the assumption be made that the government specifies

$$\dot{Y}_i/Y_i = \max\,\{\dot{Y}_i/Y_i\} = \xi_i \qquad (3\text{-}2\text{-}2)$$

as the required target growth rate for the i-th sector. Substituting (3-2-2) for $E\,J\,\dot{Y}$ in (2-3-1) gives

$$E\,b\,Y + E\,J\,\dot{Y} = E(b + J\xi)Y \qquad (3\text{-}2\text{-}1)'$$

where ξ is the $n \times n$ diagonal matrix.

The second constraint comes from capital stock considerations. In order to make explicit the role of capital imports it is necessary to assume that the domestic capital is of negligible amount as compared with the imported capital. This would be plausible in the economy under discussion which suffers from a chronic shortage of indigenous capital. This postulate will be retained throughout this chapter. Lack of inducements to invest, as pointed out by R. Nurkse[3], may prevent the total capital stock, approximated by the foreign capital K_f, from being fully utilized. To ensure the satisfaction of investors of all industries, it is only necessary for the induced investment to be no more than the rate of marginal change in the capital stock:

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[3] Problems of Capital Formation in Underdeveloped Countries, N. Y., 1966, Ch. 1, esp. pp. 5-11. His theory in this connection is in essence a revival of the market-size theory as set forth by A. Smith.

$$J\dot{Y} \leq I_f = \dot{K}_f$$

integrating which with respect to time and eliminating the resulting integral constant by appropriate selection of units we can get

$$J\,Y \leq K_f\ . \tag{3-2-3}$$

The third constraint comes from overall balance-of-payments considerations as pertinent to present-day developing economies. The maximum allowable extent of the import surplus of goods and services is postulated to be D in real terms:

$$M - X \leq D\ ,$$

i.e.

$$\underline{m}\,Y \leq D + X - \overline{m} = \{d_i\} + \{X_i(0)\ e^{\lambda i^t}\} + \{\overline{m}_i\} \qquad (\underline{m} = m - \epsilon)$$

which, on substitution of F for $D + X' - \overline{m}$, becomes

$$\underline{m}\,Y \leq F\ . \tag{3-2-4}$$

Notice that the term F is a non-linear convex function of time, which can take some numerical value in certain cases referred to above. It is also a policy parameter because D is subject to policy judgments.

Thus we are now in a position to formulate the program optimizing the net national product. The primal takes the form:

Maximize

$$E(b + J\ )Y \tag*{(3-2-1)'}$$

subject to the capital stock constraint

$$J\,Y \leq K_f\,, \qquad (3\text{-}2\text{-}3)$$

the real foreign trade constraint

$$\underline{m}\,Y \leq F \qquad (3\text{-}2\text{-}4)$$

and the non-negativity constraint

$$Y\,,\ K_f\,,\ F \geq 0\,.$$

This implies:

Maximize

$$\sum_i \left(b_i\,Y_i + \sum_j \rho_{ij}\,\xi_j\,Y_j\right)$$

subject to

$$\sum_j \rho_{ij}\,Y_j \leq K_{f,i}\,,$$

$$\underline{m}_i\,Y_i \leq F_i$$

and

$$Y_i,\ K_{f,i}\,,\ F_i \geq 0 \quad (i,j = 1,\cdots,n)\,.$$

It is obvious that the three assumptions about the solubility of the program are guaranteed here.

The maximization program above is necessarily accompanied by a minimization program as its counterpart. This program called "the dual" is based on the idea of minimizing the total value imputed socially to the foreign capital K_f and the foreign trade term F. We introduce a set of new variables z_i $(i = 1,\cdots,n)$ for K_f and z_{n+i} $(i = 1,\cdots,n)$ for F which are called "shadow prices,"

"valuations," etc. The dual now takes the form:

Minimize

$$[K_f + F]\{z\}$$

subject to

$$\begin{bmatrix} J & 0 \\ 0 & \underline{m} \end{bmatrix}' \{z\} \geq [E\ (b + J\xi)]'$$

and $z \geq 0$. This implies:

Minimize

$$\sum_i (K_{f,i}\ z_i + F_{n+i}\ z_{n+i})$$

subject to

$$\sum_i \rho_{ij}\ z_i \geq b_j\ ,$$

$$\underline{m}_j\ z_{n+j} \geq \sum_i \rho_{ij}\ \xi_j$$

and

$$z_i,\ z_{n+i},\ z_{n+j} \geq 0 \quad (i,j = 1,\cdots,n)\ .$$

The i-th shadow price z_i implies the unit price of capital imputed to the foreign capital K_f. The shadow price z_{n+i} attached to F connotes a net terms of trade. The first n constraints of the dual show that the total capital inputs required for production of the j-th output

$$\sum \rho_{ij}\ z_i$$

must be imputed at a level at least as high as the domestically produced proportion of the j-th output required for consumption with its imported proportion subtracted. The latter half of the constraints of the dual are less difficult to interpret. It implies that the money value of the import per unit of the j-th output $m_j \, z_{n+j}$ must at least be as large as the total weighted value of the capital imputs required for production of the j-th output $\sum_i \rho_{ij} \, \xi_j$. If the former is exactly equal to the latter, the duality theorem tells us that the rate of the j-th output becomes positive $Y_j > 0$.

Section 3

Linear Programming for Optimal Capital Accumulation

Capital accumulation undoubtedly plays a strategic role in economic development, especially in solving the problem of structural underemployment. For the purpose of augmenting the stock of real capital it will be useful to formulate a dynamic programming.

The main aim consists in maximizing the total flow of the increments in real capital

$$E \, J \, \dot{Y}$$

subject to the condition about the maximum sectoral growth rates (3-2-2). In other words, the primal objective function reduces now to

$$E \, J \, \xi \, Y \, ,$$

which is to be optimized subject to the full capacity limit on

operation (3-2-3) and the restraint on the foreign transactions (3-2-4). The primal and the dual are formulated as follows:

Primal:

Maximize the incremental real capital stock

$$E\ J\ \xi\ Y$$

subject to the full capacity constraint and the foreign trade constraint

$$\begin{bmatrix} J \\ \\ m \end{bmatrix} \{Y\} \leq \begin{bmatrix} K_f \\ \\ F \end{bmatrix}$$

and

$$Y \geq 0 ;$$

Dual:

Minimize the total social value of the foreign capital and the F term

$$[K_f , \quad F]\ \{z\}$$

subject to

$$\begin{bmatrix} J \\ \\ m \end{bmatrix}' \{z\} \geq [E\ J\ \xi\ ]'$$

and $z \geq 0$. The dual, if solved, gives an answer to the question: What is the minimum value of the foreign capital and of the F term, both socially valued, such that the social valuation for the unit j-th output produced domestically and abroad must at least be as high as the j-th sector's own marginal capital coefficient weighted socially. For

the implication of the dual constraint relative to the j-th sector is

$$\sum_i \rho_{ji} z_i + m_j z_{n+j} \geq \rho_{jj} \xi_j .$$

Section 4

Non-linear Programming for Optimal Growth Rate

The important target of rapid growth here is embodied in the objective function itself. For this purpose we must first transform the sectoral variables thus far used into corresponding log variables. Instead of (3-2-1) we now have

$$E \ln Y = E \ln C + E \ln I + E \ln X - E \ln M - E \ln K_f ,$$

where

$$E \ln C = E \bar{c} + E c \ln Y$$

$$E \ln I = E J \frac{d}{dt} \ln Y$$

$$E \ln M = E \ln \bar{m} + E m \ln Y .$$

Notice that

$$E J \frac{d}{dt} \ln Y$$

is the weighted sum of the sectoral growth rates, i.e.

$$E \ln I = E \frac{d}{dt} \ln Y = \sum \sum \rho_{ij} \frac{1}{Y_j} \frac{d Y_j}{d t} , \qquad (3\text{-}4\text{-}1)$$

which plays a central role in the programming following. $E \ln \bar{m}$,

$E \ln \bar{X}$, $E K_f$ and $E \ln \bar{c}$ being independent of $\ln Y$, the primal objective function can now be specified as the nonlinear function of Y :

$$E \ln Y - E\,(c - m + \epsilon) \ln Y = E\,(1 - b) \ln Y$$

where $b = c - m + \epsilon$ as before and $1 - b > 0$. This function being a positive linear combination of the concave functions of $Y = \{Y_i\}$, the present program belongs to a non-linear concave one, which can be reduced to one of finding a saddle point of the implied Lagrangean function.

Let us first consider the case in which the balance-of-payments consideration can be put aside. The capital stock constitutes the only restraint on the following primal and dual.

Primal 1 :

Maximize the concave nonlinear function of Y

$$E\,[1 - b]\,\{\ln Y\}$$

subject to the full capacity constraint

$$[J]\,\{\ln Y\} \leq \{\ln K_f\}$$

and

$$\ln Y \geq 0\,;$$

Dual 1 :

Minimize the convex linear function of z

$$[\ln K_f]\,\{z\}$$

subject to

$$[J]' \{z\} \geq E\,[1 - b]'$$

and $z \geq 0$, where z is the column vector of the shadow prices imputed to $\ln K_f$. If the j-th sector's output in log form is at less than full capacity, the social valuation for the j-th capital stock becomes zero, i.e. $z_j = 0$. If the j-th dual dummy variable is non-zero, the j-th primal choice variable becomes zero, i.e. $\ln Y_j = 0$.

By adding the second constraint from the standpoint of the balance of payments

$$\mathbf{m} \ln Y \leq \ln\,(D + X - \overline{m}) = \ln F,$$

where $D + X - \overline{m} = F$ is a kind of policy parameter as before, the programming above is modified slightly as follows:

Primal 2 :

Maximize the concave nonlinear function

$$E\,[1 - b]\,\{\ln Y\}$$

subject to

$$\begin{bmatrix} J \\ \mathbf{m} \end{bmatrix} \{\ln Y\} \leq \begin{Bmatrix} \ln K_f \\ \ln F \end{Bmatrix}$$

and $\ln Y \geq 0$;

Dual 2:

Minimize the convex linear function of z

$$[\ln K_f\,,\ \ln F]\,\{z\}$$

subject to

$$\begin{bmatrix} J \\ \cdot \\ \underline{m} \end{bmatrix}' \{z\} \leq E\ [1 - b]'$$

and $z \geq 0$, where z is the column vector of the shadow prices attached to $\ln K_f$ and $\ln F$.

An example of the Lagrangean function $L(Y,\zeta)$ relative to programming may well be shown for the expanded Primal 2:

$$L(Y,\zeta) = E\ [1 - b]\ \{\ln Y\} - \zeta \left(\begin{bmatrix} J \\ \\ \underline{m} \end{bmatrix} \{\ln Y\} - \begin{bmatrix} \ln K_f \\ \ln F \end{bmatrix} \right),$$

where ζ is the $1 \times 2n$ row vector of the Lagrangean multipliers. The saddle point of $L(Y,\zeta)$, if existent, constitutes the optimal extreme point of the primal in question.

A concrete example of solution can be given for any of the above programs. Let us exemplify such solution for the Primal 1 of this section. A two-sector developing economy is assumed to have the following coefficient matrices:

$$J = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \begin{bmatrix} 3.4 & 4 \\ 1.5 & 3 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}.$$

The parameters of capital stocks are assumed to be

$$\ln K_f = \{6.91, 4.61\}.$$

The iterative procedure of the Simplex Algorithm is applied to this program, with the optimal solution:

$$Y_1^* = e^{0.55}$$

$$Y_2^* = e^{1.26}$$

the optimal value of the maximand = 1.03 .

By way of comparison let each sectoral capital stock increase five-fold, so that

$$\ln K_f = \{8.06, 5.76\}.$$

Then cet. par. the optimal solution changes to

$$Y_1^* = e^{0.27}$$

$$Y_2^* = e^{1.89}$$

the optimal value of the maximand = 1.21 .

As slack variables are zero at the optimal extreme points, the exemplified data assure full utilization of capital stock in each sector before and after the increase of the capital imports. The optimal value of the maximand corresponding to the weighted sum of the sectoral growth rates increases from 1.03 to 1.21 by added capital imports. Thus the possibility is explicated that the increase of capital imports

is conducive to the rapid development of the underdeveloped economy[4]. It is also illustrated that not every sectoral output can be raised thereby.

Before concluding this chapter it should be added that for practical purposes the above continuous analysis can be approximated by a corresponding period analysis, in which case the technique would approach that adopted by Prof. Ichimura in dealing with intertemporal efficiency[5].

Reference may also be made to the fact that, if the above foreign-exchange balance constraint were written in money terms with the export price as a function of the volume of exports, the suggested period analysis would yield a quadratic programming, quadratic in the sense that one of the constraints is so in the independent variable. This is the line followed by H. B. Chenery and H. Uzawa in their minute analysis of a nonlinear programming of economic development in the open economy[6].

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[4]This coincides with what R. J. Ball calls the orthodoxical position about the role of capital imports in economic development. "...the analysis of this section based on the particular simple models at hand confirms the orthodoxical position that, given the propensity to save, a net inflow of capital will lead to a higher rate of growth than in the case of a balanced current account." R. J. Ball, 'Capital Imports and Economic Development: Paradoxy Or Orthodoxy,' Kyklos, Vol. XV, 1962, p. 617.

[5]ECAFE, ibid., 1960, pp. 105-110. The approximation can be done by the relation $\dot{Y}(t-1) \approx Y(t-1) - Y(t)$. Once the data of the current year are given, the optimal values of the next year are in this way predictable. For illustration, see Section 1, Chapter 5.

[6]H. B. Chenery and H. Uzawa, 'Non-linear Programming in Economic Development,' Studies in Linear and Non-linear Programming, Stanford Univ. Press, California 1958, Ch. 15.

Appendix to Chapter 3

Efficient Growth Model for an Open Economy

A variant satisfying the maximum growth criterion is furnished by the efficient growth theory, which is based on assumptions different from those of the previous sections. A time path is described as efficient when it is terminally optimal for some choice of an objective function.[1] An efficient balanced growth path will here be demonstrated to be a path with the largest growth factor among all Leontief-type balanced growth models. The balanced growth here is meant to be the growth such that the rate of growth is the same for all sectors.

Application of the efficient growth theory to a developing economy requires following assumptions to be added or specified:

(i) Consumption of each good is performed in the same proportion to total output as for every other good.

(ii) Neoclassical transformation. The implicit function between outputs and inputs are twice differentiable. Both outputs and inputs are positive. For the set of inputs fixed the output set forms a neoclassical transformation surface convex outwards. For the set of given outputs the input set forms an isoquent surface concave to the origin. Homogeneity of degree zero is subsumed.

(iii) Discrete time series. This makes a sharp contrast with the continuous time used thus far. The outputs of period t form the inputs of period $t + 1$.

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[1] K. Lancaster, Mathematical Economics, Macmillan, 1968, p. 176.

(iv) Inputs consist of foreign as well as domestic ones. The capital input is of crucial importance among the imported inputs.

(v) Some part of the outputs flows out of the economy. Some part of the inputs flow into the economy. The exports at time $t + 1$ are assumed to be in balance with the imports at time t. The imports of capital goods are expected to raise the competitive power to export in one interval in such a way as to bring the above intertemporal balance.

Notations of the Present Appendix:

y output vector

y^d vector of domestically used (or produced) output; $y^d \in y$.

y^f vector of exported output; $y^f \in y$. ($\in$ reads 'is an element of').

z input vector

z^d domestic input vector; $z^d \in z$.

z^f imported input vector; $z^f \in z$.

t time ($t = 0, \cdots, T$)

$z, y \in R^m$. (z, y is an element of real m-dimensional space).

The neoclassical transformation relationship is taken to be

$$F(y(t), z(t)) = 0.$$

The balance of outputs at period t with inpurs at period $t + 1$ is specified by

$$z^d(t + 1) - y^d(t) = 0.$$

The balance between exports at period $t + 1$ and imports at period $\underline{t}$ is set by

$$z^f(t) - y^f(t+1) = 0.$$

Subject to the above three conditions, efficiency growth requires the terminal output $y(T)$ to be maximized. The entire optimizing program for the open economy then becomes:

Maximize the terminal output

$$y(T)$$

subject to

$$F(y(t), z(t)) = 0 \quad t = 0,\cdots,T$$

$$z^d(t+1) - y^d(t) = 0 \quad t = 0,\cdots,T-1.$$

$$z^f(t) - y^f(t+1) = 0 \quad t = 0,\cdots,T-1.$$

The problem can be solved by the Lagrangean method. Let the Lagrangean be defined by use of Lagrangean multipliers λ_t, μ_t, ϕ_t, as

$$L[y(0),\cdots,y(T),\ z(1),\cdots,z(T),\ \lambda_0,\cdots,\lambda_T,\ \mu_0,\cdots,\mu_T,\ \phi_0,\cdots,\phi_T]$$

$$= y(T) - \sum_{t=0}^{T} \lambda_t\, F[y(t),\ z(t)] - \sum_{t=0}^{T-1}\sum_{i=0}^{m} \mu_{t,i}\, [z_i^d(t+1) - y_i^d(t)]$$

$$- \sum_{t=0}^{T-1}\sum_{i=0}^{m} \phi\, t,i\, [z_i^f(t) - y_i^f(t+1)] \qquad (2\text{-}4\text{-}1)$$

The first-order conditions with respect to $y_i^d(t)$, $z_i^d(t)$ are

$$\frac{\partial L}{\partial y_i^d(t)} = -\lambda_t \frac{\partial F(t)}{\partial y_i^d(t)} + \mu_{t,i} = 0 \qquad (2\text{-}4\text{-}2)$$

$$\frac{\partial L}{\partial \hat{z}_i^d(t)} = -\lambda_t \frac{\partial F(t)}{\partial z_i^d(t)} - \mu_{t-1,i} = 0 \qquad (2\text{-}4\text{-}3)$$

where $F(t)$ is short for $F[y(t),\ z(t)]$. By taking $t+1$ in (2-4-3)

instead of t, we obtain

$$\frac{\partial L}{\partial z_i^d(t+1)} = -\lambda_{t+1}\frac{F(t+1)}{z_i^d(t+1)} - \mu_{t,i} = 0. \tag{2-4-4}$$

We can eliminate $\mu_{t,i}$ between (2-4-2) and (2-4-4) to obtain

$$\lambda_{t+1}\frac{\partial F(t+1)}{\partial z_i^d(t+1)} = -\lambda_t \frac{\partial F(t)}{\partial y_i^d(t)}. \tag{2-4-5}$$

Similarly we can get for the j-th sector

$$\lambda_{t+1}\frac{\partial F(t+1)}{\partial z_j^d(t+1)} = -\lambda_t \frac{\partial F(t)}{\partial y_j^d(t)}. \tag{2-4-6}$$

Eliminating λ_t, λ_{t+1} from (2-4-5) and (2-4-6) we arrive at

$$\frac{\partial F(t+1)}{\partial z_i^d(t+1)}\bigg/\frac{\partial F(t+1)}{\partial z_j^d(t+1)} = \frac{\partial F(t)}{\partial y_i^d(t)}\bigg/\frac{\partial F(t)}{\partial y_j^d(t)}, \tag{2-4-7}$$

Thus the marginal rate of substitution (MRS) between any pair of domestically produced goods used as domestica inputs in period $t+1$ is equal to their MRS as domestically used outputs in period t.

$$\frac{\partial F(t)}{\partial z_i^f(t)}\bigg/\frac{\partial F(t)}{\partial z_j^f(t)} = \frac{\partial F(t+1)}{\partial y_i^f(t+1)}\bigg/\frac{\partial F(t+1)}{\partial y_j^f(t+1)}, \tag{2-4-8}$$

which imples that the MRS between any pair of imported goods as inputs in period t is equal to their MRS as outputs in period $t+1$. If the imported inputs consist entirely of capital goods, the above result means that the ratio of the marginal products of capital goods as inputs in period t is the same as the marginal rate of transformation in production in period $t+1$ for any pair of goods.

The conditions (2-4-7), (2-4-8) are an application of Dorfman, Samuelson, and Solow's "intertemporal efficiency conditions."[2] (2-4-7) and (2-4-8) as partial differential equations are soluble if certain integrability conditions are satisfied.

From (2-4-2) we have

$$\frac{\partial F(t)}{\partial y_i^d(t)} = \frac{\mu_{t,i}}{\lambda_t} \tag{2-4-9}$$

of which the right-hand side has the meaning of some kind of real shadow price of the i-th domestic good. To interpret λ_t we use the zero order homogeneity of the transformation $F[y(t), z(t)] = 0$ and Euler's theorem, to obtain

$$\sum \frac{\partial F(t)}{\partial y_i^d(t)} y_i^d(t) + \sum \frac{\partial F(t)}{\partial z_i^d(t)} z_i^d(t) = 0 . \tag{2-4-10}$$

If we denote the first sum on the left hand side of (2-4-10) by $V(t)$ and use (2-4-9), we get

$$V(t) \equiv \sum \frac{\partial F(t)}{\partial y_i^d(t)} y_i^d(t) = (1/\lambda_t) \sum \mu_{t,i} \, y_i^d(t)$$

which is interpreted as the real value of the domestic output in period t . In the second sum of the left-hand side of (2-4-10) we can substitute

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[2] Linear Programming and Economic Analysis, McGraw-Hill, 1958, Ch. 12.

$$\frac{\partial F(t)}{\partial z_i^d(t)} = -\frac{\mu_{t-1,i}}{\lambda_t}$$

from (2-4-3) and the dynamic assumption

$$z_i^d(t) = y_i^d(t-1)\ ,$$

to obtain

$$\sum \frac{\partial F(t)}{\partial z_i^d(t)} z_i^d(t) = -(1/\lambda_t) \sum \mu_{t-1,i} \ y_i^d(t-1)$$

$$= -(\lambda_{y-1}/\lambda_t) \sum \mu_{t-1,i}/\lambda_{t-1} \ y_i^d(t-1) = -(\lambda_{t-1}/\lambda_t) \ V(t-1).$$

From (2-4-10) we then have

$$V(t) = \frac{\lambda_{t-1}}{\lambda_t} V(t-1)$$

and

$$\frac{V(t) - V(t-1)}{V(t-1)} = \frac{\lambda_{t-1}}{\lambda_t} - 1\ .$$

This is obviously the rate of growth of the domestic output in the interval (t-1,t). Thus λ_t has proved to be closely related to the growth rate.

We now come face to face with our central issue. The efficient growth path derived above will be proven to be the balanced growth path with the highest growth rate. Consider any arbitrary balanced growth path with a growth rate g^d. Then the vectors $y^d(t)$ and $z^d(t)$ are such as to satisfy

$$F[y^d(t),\, z^d(t)] = 0$$

and

$$y^d(t) = (1+g^d)z^d(t)\,. \qquad (2\text{-}4\text{-}11)$$

Applying implicit differentiation to the former with respect to $z_i^d(t)$, and keeping in mind that g^d is a function of $y_i^d(t)$, we get

$$\frac{\partial F(t)}{\partial y_i^d(t)}\left[1 + g^d + z_i^d\ \frac{\partial(1+g^d)}{\partial z_i^d}\right] + \frac{\partial F(t)}{\partial z_i^d} = 0$$

or

$$-\ z_i^d\ \frac{\partial F(t)}{\partial y_i^d(t)}\ \frac{\partial(1+g^d)}{\partial z_i^d} = (1+g^d)\frac{\partial F(t)}{\partial y_i^d(t)} + \frac{\partial F(t)}{\partial z_i^d(t)}\,.$$

Since $z_i^d(t)$, $\partial F(t)/\partial y_i^d(t)$ can be taken as non-zero, we have

$$\frac{\partial(1+g^d)}{\partial z_i^d} = 0$$

if and only if

$$(1 + g^d)\ \frac{\partial F(t)}{\partial y_i^d(t)} + \frac{\partial F(t)}{\partial z_i^d(t)} = 0\,. \qquad (2\text{-}4\text{-}12)$$

The condition for the growth rate to be maximum is

$$\frac{\partial(1 + g^d)}{\partial z_i^d} = 0$$

for all i, so that

$$(1+g^d)\,\frac{\partial F(t)}{\partial z_j^d(t)} + \frac{\partial F(t)}{\partial z_j^d(t)} = 0\ . \qquad (2\text{-}4\text{-}13)$$

From the specification of balanced growth (2-4-11) and the dynamic assumption $y^d(t-1) = z^d(t)$, together with (2-4-12) and (2-4-13), we finally reach

$$\frac{\partial F(t+1)}{\partial z_i^d(t+1)} \Big/ \frac{\partial F(t+1)}{\partial z_j^d(t+1)} = \frac{\partial F(t)}{\partial y_i^d(t)} \Big/ \frac{\partial F(t)}{\partial y_j^d(t)}$$

which coincides with (2-4-7), the intertemporal efficiency condition. Thus the efficient balanced growth path of y^d has turned out to be the one with the highest growth rate.

Following the same method we can show that maximization of the growth rate of a balanced growth for the imported goods as input results in the intertemporal efficiency condition (2-4-8). These terminally optimal balanced growth paths are specified as the Von Neumann path.

CHAPTER IV

COUNTERCYCLICAL CAPITAL EXPORTS FOR THE STABLE GROWTH OF AN ADVANCED ECONOMY

Section 1

Introductory Observations

The primary objective of a typical underdeveloped economy consists in maximizing the level of, or the change in the level of, real national income or product. By contrast, the major concern of an advanced market economy lies in realizing secular growth without excessive cyclical instability. Therefore, stabilization policy generally aims at full employment with price stability. The growth pattern satisfying these two stabilization aims is what the so-called "stable growth" requires.

Other things being equal, stress should be placed more on growth than on price stability. The author shares in this connection with the perspicacious opinion of Sir Roy Harrod who states:

"Economic growth is the grand objective. ...Price stability is a subordinate objective. Those who hold that price stability is highly desirable... should do so because they believe that price stability is conducive to higher production, i.e. to growth."[1]

The theory of countercyclical capital exports dates from the

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[1] R. F. Harrod, Reforming the World's Money, New York, 1966, p. 78.

1930's and witnessed some vivid discussions in the 40's and the 50's. Its basic idea can be stated as follows:

Increase (or decrease) of capital exports, through changes in income, leads to increase (or decrease) of exports. This increase (or decrease) in the current account of the balance of payments tends to fill, via the multiplier effect, the deflationary (or the inflationary) gap of domestic effective demand.

This theory has indeed found more critics than supporters among economists such as N. S. Buchanan,[2] K. Zweig,[3] C. P. Kindleberger,[4] R. Nurkse.[5] Capital exports as a domestic stabilizer may remain a marginal factor as compared with usual tools of compensatory finance. The role of capital exports may also be limited by the inherent difficulty of controlling their timing, which applies especially to direct investment abroad. However, these criticisms do not invalidate an attempt to build a theoretical model for the stable growth of the advanced economy. As far as theory is concerned, development of such a model has certainly some validity and usefulness. And there is at least one economist who, based on his empirical research, states that

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[2] N. S. Buchanan, International Investment and Domestic Welfare, New York, 1945.

[3] K. Zweig, 'Strukturwandlungen und Konjunkturschwankungen im englischen Aussenhandel der Vorkriegszeit,' Weltwirtschatliches Archiv, 1929, Vol. 1.

[4] C. P. Kindleberger, International Economics, Illinois, 1953, esp. Ch. 17.

[5] R. Nurkse, Internationale Kapitalbewegungen, Wien, 1935.

"When left alone in the past, American private net investment abroad... exercised some automatic stabilizing influence on aggregate private investment and employment in the U. S."[6]

Section 2

Countercyclical Capital Exports: An Endogenous Model of Cyclical Growth in an Open Economy

The present section discusses capital exports within the framework of an endogenous model of cyclical growth. Though both A. W. Phillips and Bergstrom have presented distinct models of cyclical growth, the model to be developed here will turn out to be distinguishable from each of them, especially with respect to essential factors generating fluctuations.[7] The present ultimate aim consists in showing what role capital exports tend to play in the cyclical growth of national product or income. For this purpose let the typical advanced market economy be divided into the capital exports sector 1 and the remaining sector 2.

In a model of endogenous cyclical growth it is the behavior of capital stock which plays a strategic role. The offsetting effect of capital accumulation on income-induced investment is called the 'capital

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[6] W. P. Eagle, Economic Stabilization: Objectives, Rules and Mechanism, Princeton Univ. Press, 1952, p. 220.

[7] A. W. Phillips, 'A Simple Model of Employment, Money and Prices in a Growing Economy,' Economica, Nov. 1961, pp. 360-370. A. R. Bergstrom, 'A Model of Technical Progress, the Production Function and Cyclical Growth,' Economica, Nov. 1962, pp. 357-370.

stock adjustment principle' by R. C. O. Matthews.[8] The proponents of this kind of theory are M. Kalecki,[9] N. Kaldor,[10] K. K. Kurihara.[11]

The Model

Y in this chapter represents national income. The country's total amount of capital exports is assumed to be related to national income Y. Since the national income consists of the sum of the income of the capital exports sector Y_1 and that of the second sector Y_2, the assumption implies

$$\text{capital exports} = \bar{a}_1 + a_1(Y_1 + Y_2),$$

where $\bar{a}_1$ and a_1 are manipulative parameters.

With the column vector of the sectoral capital stock $K = \{K_i\}$ $(i = 1,2)$ and the matrix of capital coefficients $J = [\rho_{ij}]$ $(i,j = 1,2)$ introduced, the investment function $I = \{I_i\}$ now becomes

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[8] R. C. O. Matthews, The Business Cycle, Univ. of Chicago Press, 1959, esp. p. 40-43; 'Capital Stock Adjustment Theories of the Trade Cycle and the Problem of Policy,' Post-Keynesian Economics, ed. by K. K. Kurihara, London, 1962, Ch. 7.

[9] M. Kalecki, 'A Macrodynamic Theory of Business Cycles,' Econometrica, Oct. 1935.

[10] N. Kaldor, 'A Model of the Trade Cycle,' Economic Journal, March 1940.

[11] K. K. Kurihara, 'An Endogenous Model of Cyclical Growth,' Oxford Economic Papers, Oct. 1960.

$$I = J\dot{Y} - \eta K - \bar{a} - aY, ^{12} \qquad (4\text{-}2\text{-}1)$$

where $\eta = [\eta_i]$ is the diagonal matrix of the coefficients of capital stock adjustment and where $\bar{a}$ and a are

$$\bar{a} = \begin{bmatrix} \bar{a}_1 \\ 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 & a_1 \\ 0 & 0 \end{bmatrix}.$$

The other behavioral equations being assumed to be the same as in Section 2, Chapter II, the equilibrium condition now becomes

$$Y = \bar{c} + cY + J\dot{Y} - \eta K - \bar{a} - aY + X - \bar{m} - mY. \qquad (4\text{-}2\text{-}2)$$

Combining (4-2-1) and (4-2-2) results in the following system of simultaneous first-order differential equations in Y :

$$J\dot{Y} - HY = d, \qquad (4\text{-}2\text{-}3)$$

where

$$H = \begin{bmatrix} h_1 + a_1 & a_1 \\ 0 & h_2 \end{bmatrix}$$

($h_i = 1 - c_i + m_i - \epsilon_i \lesseqgtr 0 ; i = 1,2;$ h_i reduces to the marginal propensity to invest. Differentiation of $Y_i = C_i + I_i + X_i - M_i$ with respect to Y_i yields $1 = c_i + \text{MPInv}_{\cdot i} + \epsilon_i - m_i$. Hence $1 - c_i + m_i - \epsilon_i = \text{MPInv}_{\cdot i} = h_i$).

12 As is usually the case with cyclical-growth models, it is this investment function that is of central importance in analysis. To be more specific, it is the interdependence of sectoral investments that constitutes the direct cause of fluctuating elements in the process of growth.

and

$$d = \begin{bmatrix} \bar{a}_1 + n_1K_1 + \bar{m}_1 - \bar{X}_1 - \bar{c}_1 \\ n_2K_2 + \bar{m}_2 - \bar{X}_2 - \bar{c}_2 \end{bmatrix}.$$

The characteristic equation of (4-2-3)

$$|J\, r - H| = 0 \tag{4-2-4}$$

gives the characteristic roots r_1 and r_2, which are of crucial importance in determining the time paths of Y_1 and Y_2.

The present interest lies in the cyclical growth of sectoral incomes. Investigation of the quadratic equation (4-2-4) reveals that the characteristic roots become conjugate complex numbers if and only if both

$$\frac{\{\rho_{11}h_2 + \rho_{22}h_1\}\{\rho_{21} + \rho_{22}\}}{2\, h_2} > \begin{vmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{vmatrix} > 0$$

and (4-2-5)

$$a_\alpha > a_1 > a_\beta$$

are satisfied, where

$$a_\alpha = \frac{-(\rho_{11}h_2 + \rho_{22}h_1)(\rho_{21} + \rho_{22}) + 2(\rho_{11}\,\rho_{22} - \rho_{12}\rho_{21})h_2 + \sqrt{D_a}}{(\rho_{21} + \rho_{22})^2}$$

$$a_\beta = \frac{-(\rho_{11}h_2 + \rho_{22}h_1)(\rho_{21} + \rho_{22}) + 2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21})h_2 - \sqrt{D_a}}{(\rho_{21} + \rho_{22})^2}$$

and

$$D_a = [(\rho_{11}h_2 + \rho_{22}h_1)(\rho_{21} + \rho_{22}) - 2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21})h_2]^2$$

$$- (\rho_{21} + \rho_{22})^2 (\rho_{11}h_2 + \rho_{22}h_1)^2 .$$

The complementary (auxiliary) function Y_c itself will thus become

$$Y_c = \alpha_i e^{\theta t} \cos(\omega t + \epsilon_i) \quad (i = 1,2) , \qquad (4\text{-}2\text{-}6)$$

where α_i and ϵ_i are later to be definitized,[13]

$$\theta = \frac{\rho_{11}h_2 + \rho_{22}h_1 + \rho_{22}a_1 + \rho_{21}a_1}{2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21})}$$

and

$$\omega = \frac{\sqrt{2[2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21})h_2 - (\rho_{11}h_2 + \rho_{22}h_1)(\rho_{21} + \rho_{22})]a_1 \ldots}}{\ldots}$$

$$\ldots \frac{\ldots - (\rho_{21} + \rho_{22})^2 a_1^2 - (\rho_{11}h_2 + \rho_{22}h_1)^2}{2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21})} .$$

The condition for the cyclical growth of sectoral outputs (4-2-5) shall be kept throughout the following analysis of this section.

Additional inquiry shows that both incomes Y_1 and Y_2 tend to yield the following cyclical patterns around an equilibrium path to be

[13] For the term 'definitize' see A. C. Chiang, Fundamentals of Mathematical Economics, McGraw-Hill, 1967, p. 440.

specified later:

(i) explosive fluctuations if and only if

$$a_i > -\frac{\rho_1 h_2 + \rho_{22} h_1}{\rho_{21} + \rho_{22}}, \tag{4-2-7}$$

(ii) regular fluctuations if and only if

$$a_i = -\frac{\rho_1 h_2 + \rho_{22} h_1}{\rho_{21} + \rho_{22}} \tag{4-2-8}$$

and

(iii) damped fluctuations if and only if

$$-\frac{\rho_1 h_2 + \rho_{22} h_1}{\rho_{21} + \rho_{22}} > a_1 . \tag{4-2-9}$$

Next let us look for the particular integrals of (4-2-3). It is first necessary to specify the functional form of the capital stock. Let the assumption be made that

$$K = \left\{ K_i(0) e^{k_i t} \right\} \quad (i = 1,2) ,$$

where k_i is the growth rate of the capital stock of the i-th sector and $K_i(0)$ the initial value of it. This, together with the retained assumption about the exports function

$$X = \left\{ \bar{X}_i(0) e^{\lambda_i t} \right\} \quad (i = 1,2) ,$$

gives rise to a specified version of the equation system (4-2-3), i.e.

$$J\dot{Y} - HY = d \tag{4-2-10}$$

with

$$d = \begin{bmatrix} \bar{a}_1 + \eta_1 K_1(0)e^{k_1 t} + \bar{m}_1 - \bar{X}_1(0)e^{\lambda_1 t} - \bar{c}_1 \\ \eta_2 K_2(0)e^{k_2 t} + \bar{m}_2 - \bar{X}_2(0)e^{\lambda_2 t} - \bar{c}_2 \end{bmatrix}.$$

To find the particular solution of (4-2-10) we try the solution system

$$Y(t) = \left\{ \bar{B}_i + B_i e^{\delta_i t} \right\} \quad (i = 1,2) , \tag{4-2-11}$$

which implies

$$\dot{Y}(t) = \left\{ B_i \ \delta_i e^{\delta_i t} \right\}$$

and where B_i , B_i and δ_i are undetermined coefficients.

Inserting these trial solutions (4-2-11) into (4-2-10) yields:

left side of (4-2-10) $J \dot{Y} - H Y =$

$$\begin{bmatrix} (h_1 + a_1)\bar{B}_1 + a_1\bar{B}_2 + \rho_{11}B_1e^{\delta_1 t} + \rho_{12}B_2e^{\delta_2 t} \\ h_2\bar{B}_2 + \rho_{21}B_1e^{\delta_1 t} + \rho_{22}B_2e^{\delta_2 t} \end{bmatrix},$$

which must be equal identically to the right side of (4-2-10), d , regardless of the value of time t . The result is the six equations for the six unknowns $\bar{B}_1, \bar{B}_2, B_1, B_2, \delta_1$ and δ_2 . Thus the unique values of the particular solution (4-2-11) can usually be obtained. Let them be designated by $\bar{B}_i^*, B_i^*$ and δ_i^* $(i = 1,2)$. We are now able to give a

complete solution of the equation system (4-2-3) or (4-2-10). Combining the complementary function Y_c (4-2-6) with the particular integral $\overline{Y}(t)$ (4-2-11) thus specified yields the general solution:

$$Y(t) = \left\{\overline{B}_i^* + B_i^* e^{\delta_i^* t}\right\} + \left\{\alpha_i e^{\theta t} \cos(\omega t + \gamma_i)\right\} \quad (i = 1,2). \qquad (4\text{-}2\text{-}12)$$

Once the two initial conditions $Y(0)$ and $\dot{Y}(0)$, are given, it is possible to definitize α_i and γ_i. The amplitude of the sinusoidal complementary function $\alpha_i e^{\theta t}$, its phase γ_i and its period $2\pi/\omega$ can thus be specified. The final result is the completely definitized general solution (4-2-12).

The economic implication of (4-2-12) is that it satisfies Harrod's condition of warranted growth as extended into a multi-sectoral growth; it warrants continuous intended harmonization of the interests on the saving-side and the investment-side in the i-th sector of the open economy, bringing about full utilization of the current capital in the respective sectors. The first component of the general solution depicts a trend, (or an equilibrium), path of a sectoral income in the sense that it is the time path to which the general solution converges under the condition (4-2-9).[14] Further economic meanings can be attached to it with further assumptions introduced.

The national income is the sum of the net sectoral incomes.

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[14] The second component describes the deviation from the trend part of the sectoral income. This distinction is reminiscent of Milton Friedman's famous permanent income hypothesis, which states that a consumer unit's income consists of transitory and permanent components.

Suppose the j-th sector's income leads the i-th sector's by π radian. For the assumed two-sector economy the national income $Y_i(t) + Y_j(t)$ obviously shows less violent fluctuations than any sectoral income around some normal level of it, $\bar{Y}_i(t) + \bar{Y}_j(t)$, which is the sum of the sectoral particular integrals. In the opposite case where each sector has the same phase as well as the period, the national income fluctuates more violently around its equilibrium path than any sectoral income. Thus the present analysis gives a potent support to a proposition that leads or lags of one sectoral income relative to another cause cyclical fluctuations in the process of economic growth.[15]

The characteristic of the present model is that the essential factors generating cyclical growth are determined endogenously. The critical values a_α and a_β (4-2-5), the power θ and the factors determining the period ω (4-2-6) and the important elements determining the nature of fluctuations are all governed by the built-in elements: the marginal propensities to invest h_i's and the sectoral capital coefficients ρ_{ij}'s.

Perhaps it will facilitate our understanding to give a diagrammatic representation of the general solution (4-2-12) on the basis of (4-2-5) and on the assumption that the two sectors differ in phase by π radian.

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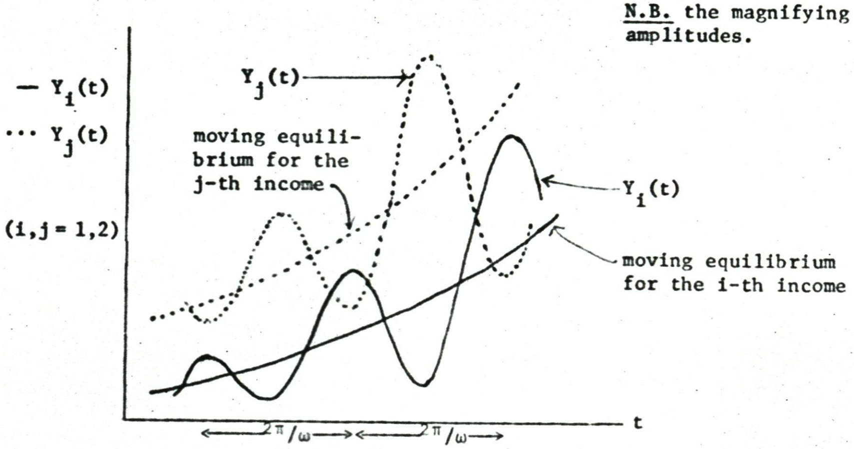
[15] cf. R. F. Harrod, Towards A Dynamic Economics, London Macmillan, 1951, p. 117.

Figure 4-2-1

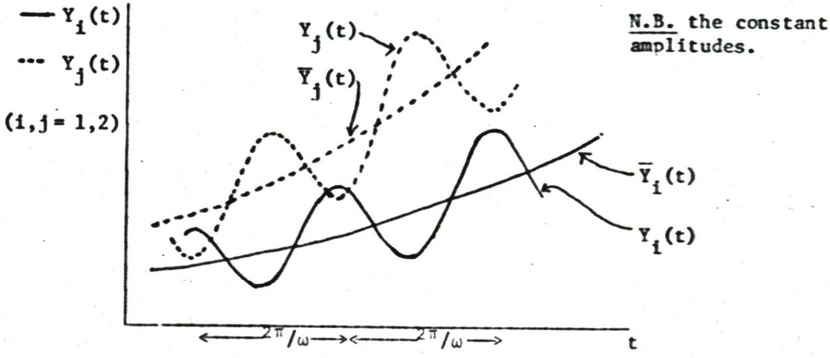
Nature of the Cyclical Growth of Sectoral Incomes

(i) (4-2-7) and $\gamma_i - \gamma_j = \pi$:



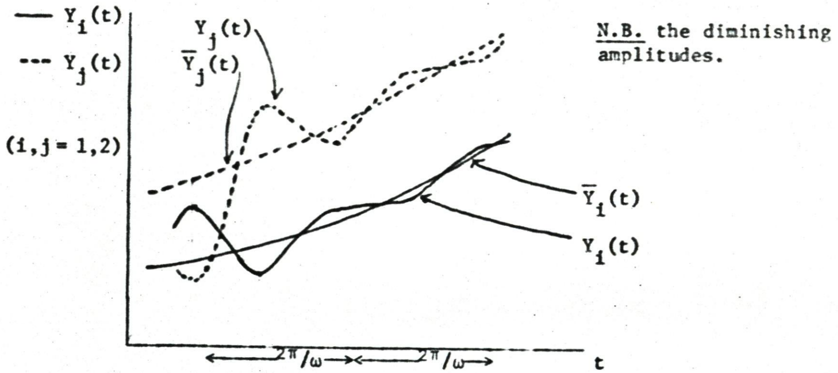
With the i-th sector leading the j-th sector by π radian the condition (4-2-7) induces the amplitude of the national income at time t to be smaller than that of any sectoral income at time t. The economy witnesses more stabilized explosive fluctuations of the national income $Y_i(t) + Y_j(t)$ around its moving equilibrium path $\overline{Y}_i(t) + \overline{Y}_j(t)$ with the magnifying amplitude.

(ii) (4-2-8) and $\gamma_i - \gamma_j = \pi$:



With the difference in phase assumed the same the condition (4-2-8) results in more stabilized regular fluctuations of the national income $Y_i(t) + Y_j(t)$ around its moving equilibrium path $\bar{Y}_i(t)+\bar{Y}_j(t)$ with the constant amplitude.

(iii) (4-2-9) and $\gamma_i - \gamma_j = \pi$:



The difference in phase being the same, the condition (4-2-9)

brings about more stabilized damped fluctuations of the national income $Y_i(t) + Y_j(t)$ around its moving equilibrium path $\overline{Y}_i(t) + \overline{Y}_j(t)$ with the diminishing amplitude. It is easy to visualize each of the three cases on the opposite assumption concerning the difference in phase, i.e. $\gamma_i = \gamma_j$.

The above analysis leads to the following conclusions:

I. The marginal propensity to export capital (the ratio of the increment of capital exports to the increment of national income), a_1, plays a crucial role in determining the time path of national income. Its regular or damped oscillations require a_1 to assume a certain value within a certain range. If a_1 is sufficiently manipulative, the policy maker will have less difficulty in attaining the stable growth desired.

II. Autonomous capital exports (the constant term in the capital exports function), $\overline{a}_1$, plays a certain role in determining the moving equilibrium path of stable growth. If $\overline{a}_1$ is flexible enough, it can also serve as a potential policy parameter.

III. The capital stock adjustment coefficient η_i also affects the moving equilibrium path of the stable growth desired. Sufficient flexibility of that coefficient will also make it easier to achieve the target growth.

IV. On the favorable assumption that leads or lags of one sectoral income relative to another are subjected to policy manipulations, the government could accomplish more stabilized fluctuations of the national income around its equilibrium path, or, in other words,

more stable cyclical growth.[16]

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[16] The author has failed to specify in rigorous terms a relationship between the above equilibrium path $\overline{Y}_1(t)$ and the full-employment path with stable prices, or between the former and the path that the increase of population and technological improvements allow. Mathematically it is due to the fact that the particular integral (4-2-11) cannot incorporate the variable of population, prices and/or labor productivity. The assumption to regard the above equilibrium path $\overline{Y}_1(t)$ as the full-employment path with stable prices or as the path corresponding to Harrod's natural growth does indeed lead to a few economically meaningful observations, but fails to meet a justification in rigorous terms. Perhaps this will betray a limitation of mathematics in economics.

CHAPTER V

CAPITAL EXPORTS AND THE OPTIMIZING MODELS FOR THE STABLE GROWTH OF AN ADVANCED ECONOMY

Section 1

Linear Programmings for Optimal Employment

A typical advanced economy is the one in which employment tends to be a function of effective demand, rather than real supply. Lack of effective demand in general becomes a cause of unemployment. One of the appropriate targets pertinent to the advanced economy is the realization of a dynamic maximization of employment with selected constraints in view. From among these constraints, focus will center on the equilibrium condition for a "steady advance" (R. F. Harrod) and the labor supply constraint. And Y in this chapter indicates national income in a multi-sectoral context.

The relation between employment and effective demand shall be specified as:

$$N_i = f(Y_i) \quad (i = 1,\cdots,n), \tag{5-1-1}$$

where N_i is the labor input of the i-th sector and Y_i its potential income. The most common approximation of the general function (5-1-1) is the first two terms of the Taylor expansion,

$$N_i = f(Y_i(0)) + f'(Y_i(0))(Y_i - Y_i(0))$$

where $f(Y_i(0))$ is a sectoral initial employment and $f'(Y_i(0))$ is

the marginal sectoral labor-income ratio at the initial period. Let $f(Y_i(0)) - f'(Y_i(0))Y_i(0)$ and $f'(Y_i(0))$ be denoted by $\bar{n}_i$ and n_i, respectively, and the above approximation becomes

$$N_i = \bar{n}_i + n_i Y_i \, . \tag{5-1-2}$$

The condition for maintaining an already high rate of growth of income on an even keel is provided by the ex-ante equalization of the investment-side and the saving-side in the present open economy. In terms of the notations in p. 36, this means

$$E\,Y = E\,\bar{c} + E\,c\,Y + E\,J\,\dot{Y} + E\,X - E\,K_x - E\,\bar{m} - E\,m\,Y \, ,$$

where K_x is the column vector of sectoral capital exports. Notice that the national income here is $E\,Y = \sum Y_i$, which is smaller than the net national product $E\,Y + E\,K_x$ by the amount of net payments made abroad on the item of autonomous capital exports $E\,K_x$.[1] In the disaggregated context the above condition implies

$$Y = \bar{c} + c\,Y + J\,\dot{Y} + X - K_x - \bar{m} - m\,Y \, . \tag{5-1-3}$$

Equation (5-1-3), if maintained, would guarantee a full utilization of current sectoral capital without excess or short capacity.

The capital exports have indeed no direct effect on income and employment, but they have indirect effects on income and employment. This is explained by the fact that exports, especially of investment goods for underdeveloped countries, are dependent on the availabiltiy of funds with which to finance them. This kind of exports characterizes the

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[1]Gerald M. Meier, International Trade and Development, Harper & Row, 1964, p. 66; The International Economics of Development, Harper & Row, 1968, pp. 69-70.

exports pattern of the typical advanced economy. W. Guth states that "We could almost call exports of investment goods a function of medium and long term capital exports."[2] Capital exports are thus a significant factor determining income and hence employment. This is especially true of export-oriented investment goods industries.

The availability of the fixed labor supply enters the system as the second important constraint on the program. The condition that labor demand in (5-1-2) cannot exceed the supply of labor $\overline{L}_i$ seems at first blush to be easily specified:

$$N_i = \overline{n}_i + n_i Y_i \leq \overline{L}_i \, .$$

This, however, omits consideration of one of the salient features common to typical advanced countries. If the full employment policy is pushed to its extreme, the actual economy will suffer from enormous strains on the labor supply and/or plant capacity leading to price inflation. The cost of price stability must be paid. The delicate balance between more income and greater price stability may render some unemployment inevitable. The rate of unemployment consistent with price stability is about 4% according to A. M. Okun,[3] and something

[2] W. Guth, Capital Exports to less Developed Countries, Dordrecht-Holland, 1963, pp. 41-46. The elasticity of the exports of heavy-industry goods to foreign assistance averages 0.2-0.3 in the case of Japan (donor) and underdeveloped economies (recipients and importers of heavy-industry goods from Japan). cf. A. Onishi, Economic Development of Underdeveloped Countries, Tokyo, 1966, p. 156.

[3] Arthur M. Okun, 'Potential GNP: Its Measurement and Significance,' Proceedings of the Business and Economic Section, The American Statistical Association, 1962, pp. 95-106.

like 5 to 6% according to P. A. Samuelson and R. M. Solow.[4] Extending Okun's concept to the corresponding multi-sectoral economy suggests the following revision, i.e.

$$N_i = \bar{n}_i + n_i Y_i \leq (1 - 0.04)\, \bar{L}_i = .96\, \bar{L}_i \qquad (5\text{-}1\text{-}4)$$

or, in matrix form,

$$N = \bar{n} + n\, Y \leq .96\, \bar{L} \qquad (5\text{-}1\text{-}4)'$$

where n is the diagonal matrix.

A few transformations are needed for us to reach a standard format of programming. The first constraint (5-1-3) will now take the form

$$J\,\dot{Y} - H\,Y = d' \qquad (5\text{-}1\text{-}3)'$$

where $J = [\rho_{ij}]$ (capital-coefficient matrix),

$H = [h_i]$ $(h_i = 1 - c_i + m_i - f_i)$ (diagonal matrix)

and $d' = \{K_{x,i} - \bar{X}_i(0)e^{\lambda_i t} + \bar{m}_i - \bar{c}_i\}$. The variables Y and $\dot{Y}$ are treated as independent of each other, as in the Calculus of Variations. Notice that the sectoral exports are of exponential form, so that the dynamic non-linear constraint d' would make the term non-linear programming appropriate to the following program. The d', however, could assume some numerical value either at a point of time or in the

[4] P. A. Samuelson and R. M. Solow, 'Analytical Aspects of Anti-Inflation Policy,' AER, May 1960, pp. 177-194. See esp. the "Modified Phillips Curve for the United States" in p. 192.

case of zero or negligible growth rates of the sectoral exports over time; in such cases, the term linear programming will retain its validity. With the linear labor constraint (5-1-4) intact, the primal and its counterpart become:

Primal:

Maximize the linear objective function π

$$\pi = 0\,\dot{Y} + E\,\bar{n} + E\,n\,Y$$

subject to the stable advance condition

$$J\,\dot{Y} - H\,Y = d'$$

the labor-supply constraint

$$0\,\dot{Y} + n\,Y \leq .96\bar{L} - \bar{N}$$

and $\dot{Y}$, $Y \geq 0$;

Dual:

Minimize the objective function

$$[d' \,,\; .96L - \bar{n}]\,\{z\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \end{bmatrix}' \{z\} \quad [0 \,,\; E\,n]'$$

and $z \geq 0$.

An example of the solution may be given. A simplified two-sector economy, with a capital-exports sector and a non-capital-exports

sector, is postulated. The capital-exports sector will enjoy larger exports of goods and services for the above-mentioned reason. This can be reflected with such an exemplified vector as[5]

$$d' = \begin{bmatrix} d'_1 \\ \\ d'_2 \end{bmatrix} = \begin{bmatrix} -8 \\ \\ -4 \end{bmatrix}.$$

In view of the fact that the expansionary effect on income and employment is larger in the first sector, the matrices on employment may, in terms of some appropriate units such as wages, be specified as

$$\bar{n} = \begin{bmatrix} 10 \\ \\ 8 \end{bmatrix}$$

and

$$n = \begin{bmatrix} .6 & 0 \\ \\ 0 & .4 \end{bmatrix}.$$

The other necessary data are assumed to be

$$L = \begin{bmatrix} 24 \\ 15 \end{bmatrix}$$

$$J = \begin{bmatrix} 2 & 1 \\ \\ 1 & 2 \end{bmatrix}$$

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[5]The larger the parameter of the capital exports, the larger becomes cet. par. the value of d'. This is the way the relative role of the capital exports in the present program can be discussed.

$$H = \begin{bmatrix} .6 & 0 \\ 0 & .6 \end{bmatrix}.$$

The optimal solution space for this primal turns out to be

$$(Y_1^*, Y_2^*, \dot{Y}_1^*, \dot{Y}_2^*; s_1^*, s_2^*, s_3^*, s_4^*) =$$

$$(21.3, 0.67, 0, 0; 0, 0, 5.04, 15.44)$$

where s_i's are the slack variables, with the optimal value of the employment function π^*

$$\pi^* = 35.30 .$$

The implied dual can also be solved in a similar way to prove the duality theorem. Exactly the same answer is derived when the partial derivatives of the corresponding Lagrangean

$$L = 18 + .6Y_1 + .4Y_2 - \eta_1 (2\dot{Y}_1 + \dot{Y}_2 - .6Y_1 + 13.04) -$$

$$\eta_2 (\dot{Y}_1 + 2\dot{Y}_2 - .6Y_2 + 9.6) - \xi_1 (.6Y_1 - 13.04) -$$

$$\xi_2 (.4Y_2 - 6.40)$$

are all made equal to zero and the resulting eight equations solved. η_1, η_2, ξ_1 and ξ_2 are the Lagrangean multipliers which reduce to the shadow prices z_1, z_2, z_3 and z_4 of the dual exemplified implicitly.

To illustrate a more practical solution to the above primal requires the underlying continuous analysis to be replaced by the corresponding period analysis. Policy makers are in general interested

in knowing the value of the objective function at the end of one period, say, a year, assuming that variables are not subject to continuous change at each point of time. The replacement is based on the approximate relation of the marginal change in income to the absolute change in income between two periods, i.e.

$$\dot{Y}(t) = Y(t) - Y(t - 1) \ .$$

Further assumptions are the same as in the above example, except for the data on d'

$$d' = \begin{bmatrix} 2 \\ 3 \end{bmatrix} .$$

which will prevent the case of degeneracy. The sectoral exports are again given some numerical values on the assumption either that they are measured as of the end of the next period, or that their growth rates are zero or almost zero. The primal turns now into the following format:

Maximize the next-period's employment

$$\Pi(t + 1) = .6Y_1(t + 1) + .4Y_2(t + 1) + 42 ,$$

subject to the steady-advance conditions[6]

[6]The inequality in the steady-advance conditions implies that all the existing capital equipments may or may not be used fully.

$$-2Y_1(t) - Y_2(t) + 1.4Y_1(t+1) + Y_2(t+1) \leq 2$$

$$-Y_1(t) - 2Y_2(t) + Y_1(t+1) + 1.4Y_2(t+1) \leq 3$$

the labor supply constraints

$$.6Y_1(t+1) \leq 13.04$$

$$.4Y_2(t+1) \leq 6.40$$

and

$$Y_1(t),\ Y_2(t),\ Y_1(t+1),\ Y_2(t+1) \geq 0 \quad (t,\ \text{time}) .$$

This intertemporal program yields the solution

$$(Y_1(t)^*,\ Y_2(t)^*,\ Y_1(t+1)^*,\ Y_2(t+1)^*;\ s_1^*,\ s_2^*,\ s_3^*,\ s_4^*) =$$

$$(117.98/7,\ 264.98/21,\ 65.20/3,\ 16;\ 0,\ 0,\ 0,\ 0),$$

with the optimal employment

$$\pi(t+1)^* = 420.08/7 = 61.47 .$$

Since the slack variables are all zero, the solution of this example implies that full employment as well as elimination of any inflationary or deflationary divergencies are sectorally guaranteed at the optimal extreme point.

The effect of a cet. par.[7] increase of capital exports on employment can be indicated by an arbitrary increase of the value of d'. Suppose d' increases from $\{2 \quad 3\}$ to $\{4 \quad 6\}$ as a reflection of the autonomous increase of all the sectoral capital exports. The outcome is a net increase of optimal employment, π^*

$$\pi^* = 64.76$$

with the solution space

$$(Y_1(t)^*, Y_2(t)^*, Y_1(t+1)^*, Y_2(t+1)^*; s_1^*, s_2^*, s_3^*, s_4^*) =$$

$$(19.3 \;,\; 0 \;,\; 25.96 \;,\; 0 \;:\; 0, 1.36, 0, 6.40\,) .$$

This example shows that underutilization of capital and unemployment of labor must result, while total employment could increase by 64.76 - 61.47 = 3.29. This is explained by the iterative process of pivoting in linear programming.[8] The increase in the constant term of the constraint leads to the further improvement of the optimal value of the objective function. And this fact applies to any other programmings of the present chapter.

The preceding two exemplified programs have incorporated the sectoral capital imports only as one of the exogenous constraints

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[7] This implies especially that the increase of capital exports is autonomous such that its effect on the induced domestic investment is zero or negligible.

[8] The economic meaning is that excessive increase of capital exports in a fully-employed economy renders some capacity idle and hence create unemployment. Net increase in total employment absorbs this new unemployment on the favorable assumption that intersectoral mobility of labor is guaranteed.

symbolized by d' . Introduction of the sectoral capital exports as explicit variables is in order. The following example aims at optimizing total employment at the end of the next period, on the assumption that the sectoral incomes at the end of the current period are known or at least estimated. Thus the model below will be useful for predictive purposes. The primal takes the format as follows:

Maximize the terminal employment of the next period

$$.6Y_1(t + 1) + .4Y_2(t + 1) + \mu_1 K_{x,1} + \mu_2 K_{x,2} + 42 + \overline{Y}_1(t) + \overline{Y}_2(t)$$

subject to the stable advance conditions

$$1.4Y_1(t + 1) + Y_2(t + 1) + K_{x,1} = d_1'$$

$$Y \quad Y_1(t + 1) + 1.4Y_2(t + 1) + K_{x,2} = d_2'$$

the labor supply constraints

$$.6Y_1(t + 1) \leq 13.04$$

$$.4Y_2(t + 1) \leq 6.40$$

and the non-negativity conditions

$$Y_1(t + 1),\ Y_2(t + 1),\ K_{x,1},\ K_{x,2} \geq 0 \ .$$

The social valuations for the capital exports, the terminal values of the current-year incomes, d'_1 and d'_2, are respectively assumed to be

$$\begin{matrix} \mu_1 = .5 \\ \mu_2 = .1 \end{matrix} , \quad \begin{matrix} \overline{Y}_1(t) = 100 \\ \overline{Y}_2(t) = 100 \end{matrix} , \quad \begin{matrix} d_1' = 31 \\ d_2' = 23 \end{matrix} .$$

With other data intact, the solution space now becomes

$$(Y_1(t+1)^*, Y_2(t+1)^*, K_{x,1}^*, K_{x,2}^* ; s_1^*, s_2^*, s_3^*, s_4^*) =$$

$$(\ 65.2/3\ ,\ 1.72/3\ ,\ 0\ ,\ 2.79/3;\ 0\ ,\ 0\ ,\ 0\ ,\ 19.89/3)$$

with the optimal employment

$$464.95/3\ .$$

In this example there must exist unemployment in the second sector at the end of the next period, though divergent tendencies will be obviated in both sectors.

Section 2

Quadratic Programmings for Optimal Employment

Monopolistic elements in commodity markets is one of the features of a typical advanced economy, which is different from the case of a typical developing economy. The monopoly in the commodity market is reflected in the negatively sloped marginal revenue curve, which, together with the concave marginal product curve, results in the concave marginal revenue product curve. The latter can be approximated by a concave quadratic function. Each entrepreneur employs labor forces

up to the point at which the demand for labor is equal to its marginal revenue product which in turn, is a function of the micro income. Solving an implied differential equation of this relationship yields a second-order labor demand function of the type

$$N_i^2 = \bar{n}_i + n_{1,i}Y_i + n_{2,i}Y_i^2 , \tag{5-2-1}$$

where $\bar{n}_i$ (autonomous employment) is an integral constant, and $n_{1,i}$ and $n_{2,i}$ are the parameters relevant to the i-th sector.[9] Transforming the right-hand side of (5-2-1) into matrix form we get

$$E\,\bar{n} + E\,n_1\,Y + E\,n_2\,Y^2 , \tag{5-2-2}$$

where $\bar{n}$, n_1 and n_2 are all diagonal matrices, and E is a unit row vector. The maximand (5-2-2) is constrained by the steady-advance condition (5-1-3), and the labor-supply restraint (5-1-4), so that the quadratic primal reduces to:

Maximize the total employment[10]

$$0\,\dot{Y} + E\,\bar{n} + E\,n_1\,Y + E\,n_2\,Y^2$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \end{bmatrix} \begin{bmatrix} \dot{Y} \\ Y \end{bmatrix} \leq \begin{bmatrix} d' \\ .96\bar{L} - \bar{n} \end{bmatrix}$$

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[9] $n_{2,i} < 0$ is one of the implications of the Kuhn-Tucker conditions to be clarified later.

[10] To be more specific, the sum of the squared sectoral employments.

and Y, $\dot{Y} \geq 0$, where Y and $\dot{Y}$ are treated as independent of each other. The contition for this non-linear general optimizing program to be soluble is specified by the Kuhn-Tucker conditions, the meaning of which is as follows: The Lagrangean function for the above primal is

$$\Phi(Y, \dot{Y}, \phi) = 0\,\dot{Y} + E\,\overline{n} + E\,n_1\,Y + E\,n_2\,Y^2 -$$

$$\phi(J\,\dot{Y} - H\,Y - d') - \phi(0\,\dot{Y} + n\,Y - .96\overline{L} + \overline{n})$$

where ϕ is the row vector of the Lagrangean multipliers. The necessary and sufficient conditions for the point $(Y^*, \dot{Y}^*, \phi^*)$ to be an optimum one are:

(a) $\hat{\Phi}_w \leq 0$ and either $\hat{\Phi}_w = 0$ or $w = 0$:

(b) $\phi \geq 0$, $\hat{\Phi}_\phi \leq 0$ and either $\phi = 0$ or $\hat{\Phi}_\phi = 0$,

where $w = Y, \dot{Y}$, $\hat{\Phi}_{w(\phi)}$ is the column vector of the partial derivatives of Φ with respect to w (ϕ) .

The sectoral capital exports are so far treated only as potential policy parameters. When the role of capital exports is brought to light, the vector of capital exports, K_x enters the program as a set of independent variables, with the revised quadratic primal:

Maximize

$$0\,\dot{Y} + E\,\overline{n} + E\,n_1\,Y + E\,n_2\,Y^2 + E\,\gamma\,K_x$$

subject to

$$\begin{bmatrix} J & -H & 1 \\ \\ 0 & n & 0 \end{bmatrix} \begin{bmatrix} \dot{Y} \\ Y \\ K_x \end{bmatrix} \leq \begin{bmatrix} d \\ \\ .96\bar{L} - \bar{n} \end{bmatrix}$$

and $Y, \dot{Y}, K_x \geq 0$,

where $d = -\bar{X} + \bar{m} - \bar{c}$ and γ is a diagonal matrix of social valuations attached to the sectoral capital exports. The solution of this program gives a required amount of the sectoral capital exports for optimal employment in the monopolistic advanced economy. Application of the Kuhn-Tucker conditions is capable of being made in the same way.

Section 3

Linear Programmings for Optimal Consumption

A typical advanced economy may find itself in an age of high mass consumption. "A society like the United States" is "structurally committed to a high-consumption way of life."[11] Consumption tends to come to the fore as an objective of such a steady growth society. For this reason, then, this section focuses attention on the consumption function.

The maximand specified as the aggregate consumption, minus the aggregate autonomous consumption,

[11] W. W. Rostow, The Stages of Economic Growth, Cambridge, 1960, p. 81.

$$E\,c\,Y$$

is subject to the steady-advance condition and the needed prevention of overemployment. Straightforward formulation of the primal and the dual is:

Primal 1:

Maximize the aggregate induced consumption

$$0\,\dot{Y} + E\,c\,Y \tag{5-3-1}$$

subject to the condition for forestalling inflationary divergencies

$$J\,\dot{Y} - H\,Y = d' \qquad (5\text{-}1\text{-}3)'$$

the condition for preventing any overemployment

$$n\,Y \leq .96\overline{L} - \overline{n} \qquad (5\text{-}1\text{-}4)'$$

and

$$Y,\ \dot{Y} \geq 0\ .$$

Dual 1:

Minimize

$$[d',\ .96\overline{L} - \overline{n}]\ \{z\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \end{bmatrix}' \{z\} \geq [0,\ E\,c]'$$

and $z \geq 0$, where z is the column vector of shadow prices to the exogenous elements d' and the full employment level $.96\bar{L} - \bar{n}$.

A variant of the above can be obtained by combining (5-3-1) and (5-1-3)' and eliminating not directly relevant variables,

Primal 2:

Maximize the induced consumption as direct function of Y and $\dot{Y}$

$$-E\,J\,\dot{Y} + E(1 - m)Y$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \end{bmatrix} \begin{bmatrix} \dot{Y} \\ Y \end{bmatrix} \leq \begin{bmatrix} d' \\ .96\,L - \bar{n} \end{bmatrix}$$

and

$$Y,\ \dot{Y} \geq 0\ ;$$

Dual 2:

Minimize

$$[d',\ .96\,L - \bar{n}]\,\{z\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \end{bmatrix}' \{z\} \geq [-\,E\,J,\ \ E(1 - m)]'$$

and $z \geq 0$. The major difference between the two duals lies in the lower limits of the constraints.

Part of the consumer goods is provided by foreign producers. When the "international demonstration effect" (the band-wagon effect and the Veblen effect à la H. Leibenstein)[12] is strong enough, the result in foreign transactions may be excessive importation of goods and services. The foreign trade constraint specifying a maximum allowable current-account deficit comes to be an appropriate restraint to be imposed on the programming model. Since in the standard linear program the number of constraints may bear any relation to the number of variables, the introduction of this third constraint offers no technical difficulty. It simply adds to the realistic value of programming. The real foreign trade constraint is specifiable as:

$$M - X \leq D,$$

i.e.

$$\underline{m}\, Y \leq D + X - \bar{m} \qquad (\underline{m} = m - \epsilon)$$

which, on substitution of F for $D + X - \bar{m}$, becomes

$$\underline{m}\, Y \leq F,$$

where D depicts the maximum permissible extent of deficit on current account. Primal 1 and Dual 1 come now to take the revised format:

Primal 3:

Maximize the aggregate induced consumption

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[12] H. Leibenstein, "The Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand," The Quarterly Journal of Economics, May 1950, esp. Sec. II.

$$[0,\ E\ c]\ \{\dot{Y},\ Y\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \\ 0 & \underline{m} \end{bmatrix} \begin{bmatrix} \dot{Y} \\ \\ Y \end{bmatrix} \leq \begin{bmatrix} d' \\ .96L - \bar{n} \\ F \end{bmatrix}$$

and $Y,\ \dot{Y} \geq 0$;

Dual 3:

Minimize

$$[d',\ .96L - \bar{n},\ F]\ \{z\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \\ 0 & \underline{m} \end{bmatrix}' \{z\} \geq [0,\ E\ c]'$$

and

$$z \geq 0.$$

In a similar way Primal 2 and Dual 2 can be transformed into Primal 4 and Dual 4:

Primal 4:

Maximize the induced consumption as direct function of Y and $\dot{Y}$

$$[-E\ J,\ E(1 - m)]\ \{Y,\ \dot{Y}\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \\ 0 & m \end{bmatrix} \begin{bmatrix} \dot{Y} \\ \\ Y \end{bmatrix} \leq \begin{bmatrix} d' \\ .96L - \bar{n} \\ F \end{bmatrix}$$

and

$$\dot{Y}, \quad Y \geq 0 ;$$

Dual 4:

Minimize

$$[d', .96L - \bar{n}, F] \{z\}$$

subject to

$$\begin{bmatrix} J & -H \\ 0 & n \\ 0 & m \end{bmatrix}' \{z\} \geq [-E J, E(1 - m)]'$$

and

$$z \geq 0 .$$

The preceding four primals and duals have treated the sectoral capital exports as one of the exogenous constraints imposed upon the programmings. Although their role as potential policy parameter is not slighted, programming the capital exports as explicit variables is in order. Addition of this independent variable vector requires corresponding changes in the above programs, which may be illustrated in the most significant one, Primal 4 and Dual 4, as Primal 4-1 and Dual 4-1:

Primal 4-1:

Maximize the induced consumption as direct function of Y and $\dot{Y}$ with the explicit role of the sectoral capital exports in view

$$[-E\ J,\ E(1 - m),\ E\ \mu]\ \{Y,\ \dot{Y},\ K_x\}$$

subject to the steady-advance condition, the full employment ceiling and the foreign trade constraint

$$\begin{bmatrix} J & -H & 1 \\ 0 & n & 0 \\ 0 & \underline{m} & 0 \end{bmatrix} \begin{bmatrix} \dot{Y} \\ Y \\ K_x \end{bmatrix} \leq \begin{bmatrix} d \\ .96L - \overline{n} \\ F \end{bmatrix}$$

and

$$Y,\ \dot{Y},\ K_x \geq 0\ ,$$

where $d = -X + \overline{m} + \overline{c}$ and μ is a diagonal matrix of the social valuations attached to the sectoral capital exports;

Dual 4-1:

Minimize

$$[d,\ .96L - \overline{n},\ F]\ \{z\}$$

subject to

$$\begin{bmatrix} J & -H & 1 \\ 0 & n & 0 \\ 0 & \underline{m} & 0 \end{bmatrix}' \{z\} \geq [-E\ J,\ E(1 - m),\ E\ \mu]'$$

and

$$z \geq 0.$$

The above programs can be easily extended to the case in which a per-capita, rather than aggregated, consumption function is an objective. And, when some sort of social utility function is assumed to exist without regard to certain controversial difficulties, programming will assume the format of optimizing, subject to similar constraints, a general objective function of utility, social or individual, which is again a function of consumption, social or per-capita.

The conclusion drawn from the numerical example of Section 1 can be generalized. Any increase in autonomous capital exports raising the constant term of the steady-advance constraint, leads to a further improvement of the optimal value of a target function. That is, the optimal value of employment and consumption can further be improved by autonomous augmentation of capital exports. Any increased capital exports, while guaranteeing the equilibrium between the saving and the investment side, tends to lead to further exports of investment goods. This results in more income and more consumption as well as more labor demand satisfying a labor-supply limit.

CHAPTER VI

A TWO-COUNTRY MODEL:

ADVANCED-UNDERDEVELOPED WORLD ECONOMY

Section 1

Introductory Observations

The analysis so far has treated an underdeveloped and a developed economy separately for methodological reasons. The next step in the logical order of argument must be to integrate both economies by focusing on their interdependence. The capital-rich advanced economy influences the underdeveloped economy through its capital exports as well as its foreign trade; the capital-poor underdeveloped economy in turn acts on the developed economy via its capital imports as well as its foreign transactions. For instance, the provision of long-term funds for capital exports and the question of their utilization in the capital importing countries are so closely interconnected that they could hardly be treated separately.[1]

The conventional approach to the problem of interdependence of two economies is the foreign trade multiplier analysis. However ramified and sophisticated this analysis might look through consideration of

[1] W. Guth, Capital Exports to Less Developed Countries, Dordrecht-Holland, 1963, p. IX.

repercussions, extension into multi-sectoral or multi-country systems,[2] their essential feature remains unchanged; that is, they are all static analyses. Even the recent paper of E. M. B. de Dagum entitled 'Le multiplicateur dynamique d'exportation: un modèle pour l'Argentinè'[3] is in essence an extension of the static multiplier analysis in that dynamic elements enter his system for the trivial reason that his consumption and import functions are the functions of income lagged one period: $C_t = c\, Y_{t-1}$ and $M_t = m\, Y_{t-1}$ (op. cit. p. 95).

'Multiplier analysis,' or analysis of interdependent economies, is to be discussed within a truly dynamic framework of economic growth. This is our proposition in this chapter: attention is paid to the interaction of different inherent growth potentialities in the world composed of an economy capable of growing rapidly and the one structurally destined for stable growth.

The previous chapters have focused, among other things, on the

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[2] The standard works on this theory are: Fritz Machlup, International Trade and the National Multiplier, Philadelphia, 1950; F. D. Holzmann and Arnold Zellner, 'The Foreign-Trade and Balanced-Budget Multiplier,' AER, March 1958; Charles Kindleberger, International Economics, Homewood, 1953, Ch. 10 and App. E; H. G. Johnson, International Trade and Economic Growth, Harvard Univ. Press, 1961, esp. Ch.s VII and VIII, etc. All of these writers treat repercussions. The multi-country extension of the theory is seen in; H. G. Johnson, op. cit. Ch. VIII; L. A. Metzler, 'A Multi-Region Theory of Income and Trade,' Econometrica, Vol. 18, Oct. 1950; D. Dosser, 'National-Income and Domestic-Income Multipliers and their Application to Foreign-Aid Transfers,' Economica, Feb. 1963, esp. Sec. II; etc.

[3] Économique Appliqué, Genève, Tome XXII-1969, N° 1-2, pp. 89-111. Mr. Dagum's analysis has on examination turned out to be based on F. Machlup, op. cit., App. A, pp. 219-223.

specification of required capital movements from respective national viewpoints. This chapter places stress more on various *effects* of long-term capital movements upon the macro behavior of each open economy, though general causes or determinants of such capital movements continue to remain to be solved.

The methodology of comparative dynamics helps fulfill our objective. The national outputs and their growth rates in the open system with international capital movements are compared with those in the same system devoid of such capital transactions in order to clarify comparative macro effects of such capital movements within two different national economies. And the theory suitable for this objective can also and still be found in the tradition of Harrod-Domar theory rather than the neo-classical growth theory.[4]

Section 2

The Dynamic Interdependence of an Advanced and an Underdeveloped Economy

Let the world be made up conceptually of two economies, advanced and underdeveloped, so that exports and imports of the one constitute,

[4] A two-sector or a two-country model based on optimum growth theory is being developed extensively by neoclassical economists. See, for instance, K. Hamada, 'Economic Growth and Long-Term International Capital Movements,' *Yale Economic Papers*, Spring, 1966, which deals with mobile capital; H. Uzawa, 'On a Two-Sector Model of Economic Growth,' *Review of Economic Studies*, Oct. 1961, which handles mobile capital and labor.

respectively, imports and exports of the other. The assumption is also made that autonomous capital exports from the advanced economy, the sole capital exporting country, is equal in value to autonomous capital imports (saving from abroad) of the underdeveloped economy, the sole capital importing country.

The dynamic equilibrium conditions of the advanced economy A (the suffix A below suggests 'A'dvanced) and the underdeveloped economy B (the suffix B below hints 'B'ackward) are respectively specifiable as[5]

$$Y_A - C_A + M_A = I_A + I_f + X_A, \text{ or}$$

$$Y_A = C_A + I_A + I_f + X_A - M_A , \qquad (6\text{-}2\text{-}1)$$

where

$C_A = c_A Y_A$

$I_A = \rho_A \dot{Y}_A$ (domestic induced investment)

$I_f = \overline{I}_f(t)$ (net autonomous investment abroad as a function of time)

$X_A = M_B = m_B Y_B$

and

$M_A = m_A Y_A$,

[5] (6-2-1) implies that total net expenditure (including net expenditure made abroad $X_A - M_A$), $C_A + I_A + X_A - M_A$, is smaller than national income or output Y_A by the investment made abroad I_f. (6-2-2) means that total net expenditure, $C_B + I_B + X_B - M_B$, exceeds the national income or output, the balance being financed by the imported saving (capital) S_f. The author is indebted to Prof. Cohn in these formulations.

and as

$$S_f + (Y_B - C_B) + M_B = I_B + X_B \text{ , or}$$

$$Y_B = C_B + I_B + X_B - M_B - S_f \text{ ,} \qquad (6\text{-}2\text{-}2)$$

where

$$C_B = c_B Y_B$$

$$I_B = \rho_B \dot{Y}_B$$

$$X_B = M_A = m_A Y_A$$

$$M_B = m_B Y_B$$

$S_f = \overline{S}_f(t)$ (autonomous saving from abroad as a function of time)

$Y_B - C_B$ is the domestic saving of the country B .

By assumption we have the equivalence of the autonomous investment abroad of the country A to the autonomous saving from abroad of the country B , i.e.

$$I_f = \overline{I}_f(t) = \overline{S}_f(t) = S_f \text{ ,}$$

where the term 'autonomous' is intended to mean

$$\frac{d\,\overline{I}_f(t)}{d\,Y_A} = \frac{d\,\overline{S}_f(t)}{d\,Y_B} = 0 \text{ .}$$

Performing appropriate operations on the above two equilibrium conditions leads to the transformed basic conditions as follows:

$$\rho_A \dot{Y}_A - h_A Y_A + m_B Y_B = -\bar{I}_f \qquad (6\text{-}2\text{-}3)$$

and

$$\rho_B \dot{Y}_B - h_B Y_B + m_A Y_A = \bar{S}_f , \qquad (6\text{-}2\text{-}4)$$

where

$$h_A = 1 - c_A + m_A$$

and

$$h_B = 1 - c_B + m_B .$$

Respective differentiation of (6-2-3) and (6-2-4) with respect to time results in

$$\rho_A \ddot{Y}_A - h_A \dot{Y}_A + m_B \dot{Y}_B = -\dot{I}_f \qquad (6\text{-}2\text{-}5)$$

and

$$\rho_B \ddot{Y}_B - h_B \dot{Y}_B + m_A \dot{Y}_A = \dot{S}_f . \qquad (6\text{-}2\text{-}6)$$

From the operation (6-2-3) × m_A - (6-2-6) × ρ_A comes

$$m_A h_A Y_A = -\rho_A \rho_B \ddot{Y}_B + \rho_A h_B \dot{Y}_B + m_A m_B Y_B + m_A I_f + \rho_A \dot{S}_f . \qquad (6\text{-}2\text{-}7)$$

Eliminating Y_A through combination of (6-2-4) with (6-2-7) results in the second-order differential equation of Y_B :

$$\rho_A \rho_B \ddot{Y}_B - (\rho_A h_B + \rho_B h_A)\dot{Y}_B + (h_A h_B - m_A m_B)Y_B =$$

$$m_A I_f - h_A S_f + \rho_A \dot{S}_f \, . \tag{6-2-8}$$

Following the same method we reach the counterpart for Y_A :

$$\rho_A \rho_B \ddot{Y}_A - (\rho_A h_B + \rho_B h_A)\dot{Y}_A + (h_A h_B - m_A m_B)Y_A =$$

$$h_B I_f - \rho_B \dot{I}_f + m_B S_f \, . \tag{6-2-9}$$

The complementary functions corresponding to the second-order differential equations (6-2-8) and (6-2-9) are respectively

$$\ddot{Y}_B - u \dot{Y}_B + v Y_B = 0 \tag{6-2-10}$$

and

$$\ddot{Y}_A - u \dot{Y}_A + v Y_A = 0 \, , \tag{6-2-11}$$

where

$$u = \frac{\rho_A h_B + \rho_B h_A}{\rho_A \rho_B} \quad (0 < u < 1)$$

and

$$v = \frac{h_A h_B - m_A m_B}{\rho_A \rho_B} \quad (0 < v < 1) \, .$$

The characteristic equations are common to the above two homogeneous equations. They assume the form

$$r^2 - u r + v = 0 \, .$$

The discriminant of this quadratic function of r reduces to

$$u^2 - 4v = \frac{1}{(\rho_A \rho_B)^2} \left\{ (\rho_A h_B - \rho_B h_A)^2 + 4 m_A m_B \rho_A \rho_B \right\} > 0 ,$$

which implies that the national outputs of both countries display monotonous movement over time. The general solutions of (6-2-8) and (6-2-9) can thus be written respectively as

$$Y_B = \overline{Y}_B + \beta_1 e^{r_1 t} + \beta_2 e^{r_2 t} \qquad (6\text{-}2\text{-}12)$$

and

$$Y_A = \overline{Y}_A + \alpha_1 e^{r_1 t} + \alpha_2 e^{r_2 t} , \qquad (6\text{-}2\text{-}13)$$

where $\overline{Y}_B$ and $\overline{Y}_A$ are the respective particular solutions, the equilibrium values of the respective national outputs,[6] satisfying (6-2-8) and (6-2-9),

$$r_1 = \frac{1}{2} \left(u - \sqrt{u^2 - 4 v} \right) ,$$

$$r_2 = \frac{1}{2} \left(u + \sqrt{u^2 - 4 v} \right) ,$$

[6] The remaining complementary functions are considered as deviations from the respective trend values of national outputs, i.e. $\overline{Y}_A$ and $\overline{Y}_B$. (6-2-12) and (6-2-13) can be applicable to many other problems. For example, the difference $Y_A - Y_B$ shows how the gap between the advanced and the underdeveloped economy changes over time.

$$0 < r_1 < r_2 < 1 \, ,$$

$$\alpha_1 = \frac{Y_A(0)\, r_2 - \dot{Y}_A(0)}{r_2 - r_1} \, ,$$

$$\alpha_2 = \frac{Y_A(0)\, r_1 - \dot{Y}_A(0)}{r_1 - r_2} \, ,$$

$$\beta_1 = \frac{Y_B(0)\, r_2 - \dot{Y}_B(0)}{r_2 - r_1} \, ,$$

$$\beta_2 = \frac{Y_B(0)\, r_1 - \dot{Y}_B(0)}{r_1 - r_2}$$

and $Y_i(0)$, $\dot{Y}_i(0)$ $(i = A,B)$ are respectively the initial values of $Y_i(t)$ and $\dot{Y}_i(t)$.

On the plausible assumption that the dominant root is either r_1 or r_2, we can show that the growth rates of Y_B and Y_A become eventually the same. This observation is reminiscent of the conclusion of Hans Brems.[7]

Dynamic interdependence of the two economies shall be exemplified by a unilateral shift of the marginal propensity to import and the one

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[7]'The Foreign Trade Accelerator and the International Transmission of Growth,' Output, Employment, and Growth, Harper & Brothers, N. Y., 1959, Ch. 24. He assumes the same overall capital coefficient for two economies and uses difference-equation models. Thus, his model is designed for two technologically equal economies and lacks in generality, as compared with ours.

of capital movements. First, let the underdeveloped economy witness a slight increase in its marginal propensity to import m_B for some reason. The greater m_B lowers the larger characteristic root r_2 and raises the smaller one r_1 through increased v. The discrepancy between r_1 and r_2 being changed, β_1 and β_2 must be changed, too, upwards or downwards depending on the given conditions. So the solution of the complementary function (6-2-10) must be changed along with additional factors. The particular integral $\overline{Y}_B$ must also be subject to the corresponding change (cf.(6-2-8)). Therefore the national output of the underdeveloped economy, which is (6-2-12), must proceed through a time path modified more or less in every term. The influence of the change of m_B does not limit itself to the economy B ; it extends to the economy A through the latter's export expansion. The induced changes of r_1 and r_2 result in the corresponding shifts of the coefficients α_1 and α_2, upwards or downwards depending on the initial conditions. The solution of the complementary function relative to the economy A (6-2-11) must be changed accordingly. The particular integral $\overline{Y}_A$ is also thereby influenced (cf.(6-2-9)). In the final analysis the initial unilateral change must bring about a totally different time path through which the advanced economy grows (6-2-13).

Second, let us exemplify the dynamic interdependence via a unilateral change in capital movements. Suppose that capital exports from the advanced economy increased. Then the particular solution $\overline{Y}_A$ satisfying (6-2-9) must undergo a corresponding change, larger or smaller depending on the nature of the particular integral. The result

is a modification of the overall time path Y_A given by (6-2-13). This process being viewed from the reverse side, the implied simultanwous increase of the capital inflow into the underdeveloped economy must induce a certain change in the trend value of output $\overline{Y}_B$ satisfying (6-2-8) and hence a corresponding shift of the dynamic macro behavior of the national output of the underdeveloped economy (6-2-12).

Section 3

The Dynamic Effect of Exports on an Advanced and an Underdeveloped Economy

This section is intended for revealing those truly dynamic factors which are excluded from the so-called 'dynamic' foreign trade multiplier analysis mentioned above. Attention is thus focused on that reciprocal of the marginal ratio of exports to output which corresponds to the marginal export multiplier of the conventional static analysis.[8]

The assumption is first made that the particular solutions $\overline{Y}_A$ and $\overline{Y}_B$, the equilibrium levels of national outputs, of the advanced and the underdeveloped economy, respectively, rise at the respective constant rates ζ_A and ζ_B, i.e.

[8] This section is partly stimulated by the major empirical conclusion of Robert F. Emery: "it would appear from the data that relatively high rates of economic growth are likely to follow from relatively high rates of export growth." 'The Relation of Exports and Economic Growth' Kyklos, Vol. XX, 1967, p. 484.

$$\frac{\dot{\overline{Y}}_A}{\overline{Y}_A} = \zeta_A \text{ (constant) and } \frac{\dot{\overline{Y}}_B}{\overline{Y}_B} = \zeta_B \text{ (constant).} \qquad (6\text{-}3\text{-}1)$$

Combining (6-3-1) with (6-2-3) and (6-2-4) we obtain

$$\rho_A\zeta_A\overline{Y}_A - h_A\overline{Y}_A + m_B\overline{Y}_B = \overline{I}_f \qquad (6\text{-}3\text{-}2)$$

$$\rho_B\zeta_B\overline{Y}_B - h_B\overline{Y}_B + m_A\overline{Y}_A = -\overline{S}_f \ . \qquad (6\text{-}3\text{-}3)$$

With the above assumed independence of capital movements of the two national outputs in view, we differentiate (6-3-2) with respect to $\overline{Y}_B$ to obtain

$$(\rho_A\zeta_A - h_A)\frac{\partial\overline{Y}_A}{\partial\overline{Y}_B} + m_B = 0$$

which, on the plausible assumption that $\rho_A\zeta_A - h_A \neq 0$, leads to

$$\kappa_A \equiv \frac{\partial\overline{Y}_A}{\partial\overline{X}_A} = \frac{\partial\overline{Y}_A}{\partial(m_B\overline{Y}_B)} = \frac{1}{h_A - \rho_A\zeta_A} \ . \qquad (6\text{-}3\text{-}4)$$

Here $\overline{X}_A$ is the equilibrium level of exports of the country A corresponding to $\overline{Y}_B$. Following the same procedure we arrive at

$$\kappa_B \equiv \frac{\partial\overline{Y}_B}{\partial\overline{X}_B} = \frac{\partial\overline{Y}_B}{\partial(m_A\overline{Y}_A)} = \frac{1}{h_B - \rho_B\zeta_B} \ , \qquad (6\text{-}3\text{-}5)$$

where $\overline{X}_B$ is the equilibrium level of exports of the country A corresponding to $\overline{Y}_A$ and $\rho_B\zeta_B - h_B$ is assumed to be non-zero.

(6-3-4) exactly symbolizes a trend marginal effect of exports upon the national output of the advanced economy, or what might otherwise be called the long-run 'export multiplier' of a growing advanced economy. (6-3-5) gives a measure of the long-run relative contribution of exports to the national output of a developing underdeveloped economy.[9]

Instead of making up hypothetical examples, the author tries to estimate the values of κ_A and κ_B empirically. Because of the inherent difficulty of estimating the long-run values of the coefficients, esp. ζ_i $(i = A, B)$, the results derived below can give only a very rough approximation of the true values of κ_i $(i = A, B)$.

<u>Empirical Example I</u> : <u>U.S.A. and Argentina</u>

<u>Data for U.S.A.</u> (the country A):

$\rho_A = 6.3$	(1950-1958)	S. Ichimura, *The Japanese Economy in the World*, 1965, Tokyo, p. 99.
$\zeta_A = .04$	(1860-1954)	the average annual growth rate of NI in the long-run period considered derived from: S. Ichimura, *op. cit.* p. 15.

[9] On the plausible assumption that the marginal propensity to save of the advanced economy is sufficiently larger than that of the underdeveloped economy such that $s_A > s_B$ and $h_A > h_B$, and that the capital-output ratio is larger in the underdeveloped economy than in the advanced economy so that $\rho_B > \rho_A$, we can show that $h_A - \rho_A\zeta_A > h_B - \rho_B\zeta_B$ if $\zeta_A = \zeta_B$. This means that κ_B tends to become negative more easily for the underdeveloped economy than κ_A does for the advanced economy.

$c_A = .68$	(1950-1958)	the result of regression analysis for the period considered derived from: E. M. B. de Dagum, *op. cit.* p. 102.
$m_A = .02$	(1950-1958)	derived in the same way from: E. M. B. de Dagum, *op. cit.* p. 103
$h_A = .34$	(1950-1958)	
$s_A = .32$	(1950-1958)	

Data for Argentina (the country B):

$\rho_B = 10.67$		derived from: OECD, *Quantitative Models As An Aid To Development Assistance Policy*, Paris, 1967, p. 65.
$\zeta_B = .03$	(1953-63)	the average annual rate of growth of GNP for the period considered covering a longest interval in which the GNP data of Argentina are available. Source: R. F. Emery, 'The Relation of Exports and Economic Growth,' *Kyklos*, Vol. XX, 1967, p. 477.
$c_B = .64$	(1950-1958)	Source: E. M. B. de Dagum, *op. cit.* p. 102.
$m_B = .39$	(1950-1958)	Source: E. M. B. de Dagum, *op. cit.* p. 103.
$h_B = .75$	(1950-1958)	
$s_B = .36$	(1950-1958)	

Final Results:

$$\kappa_A = \frac{\partial \bar{Y}_A}{\partial \bar{X}_A} = 11.36$$

$$\kappa_B = \frac{\partial \bar{Y}_B}{\partial \bar{X}_B} = 2.32$$

Empirical Example II : U.K. and Argentina

Data for U.K. (the country A'):

$\rho_{A'} = 6.4$ (1958-1962) Source: S. Ichimura, *op. cit.* p. 99 .

$\zeta_{A'} = .02$ (1860-1954) Source: S. Ichimura, *op. cit.* p. 15 .

$c_{A'} = .74$ (1950-1958) Source: E. M. B. de Dagum, *op. cit.* p. 102 .

$m_{A'} = .11$ (1950-1958) Source: E. M. B. de Dagum, *op. cit.* p. 103 .

$h_{A'} = .74$ (1950-1958)

$s_{A'} = .26$ (1950-1958)

Data for Argentina (the country B'):

Source: the same as in Empirical Example I.

Final Results:

$$\kappa_{A'} = \frac{\partial \bar{Y}_{A'}}{\partial \bar{X}_{A'}} = 1.62$$

$$\kappa_{B'} = \frac{\partial \overline{Y}_{B'}}{\partial \overline{X}_{B'}} = 2.32$$

This leads then to the following tentative conclusions:

1) The U.S.A. could enhance the equilibrium level of its national output by its exclusive exports to Argentina considerably and far more than Argentina could by its exclusive exports to the U.S.A.

2) The U.K. could increase the trend value of its national output by its exclusive exports to Argentina more than twice less than Argentina could by its exclusive exports to the U.K.

3) Argentina, while trying to increase its equilibrium national output, should have a proclivity to export more to the U.K. than to the U.S.A.

Section 4

The Dynamic Effects of Capital Movements upon National Outputs of an Advanced and an Underdeveloped Economy

It is the aim of this section to bring out the dynamic effects of capital exports and capital imports upon the national output of the advanced economy and that of the underdeveloped economy, respectively. For this purpose a comparison is first made between the growth path of output of the advanced economy engaged in positive capital exports and that of the advanced economy undertaking no such capital exports.

The national output of the advanced country engaged in no capital exports coincides with the solution of the complementary function (6-2-11), so that the corresponding national output denoted by Y_A' becomes

$$Y_A' \text{ (with no capital exports) } = \alpha_1 e^{r_1 t} + \alpha_2 e^{r_2 t} .$$

Therefore, the net difference of the national output with capital exports from that without capital exports reduces to

$$Y_A - Y_A' = \overline{Y}_A + \alpha_1 e^{r_1 t} + \alpha_2 e^{r_2 t} - \left(\alpha_1 e^{r_1 t} + \alpha_2 e^{r_2 t}\right) = \overline{Y}_A .$$

Our problem for the advanced economy boils now down to determination of the value and the sign of the marginal change in the above net difference of output in response to an increase of capital exports.

Likewise the major problem for the underdeveloped economy is condensed by determination of the extent to which an extra change of the equilibrium level of national output results from a unilateral increase in capital imports.

Assuming the constant growth rate of $\overline{Y}_A$ and $\overline{Y}_B$ again, we combine (6-3-1) with (6-2-3) and (6-2-4) to obtain

$$(\rho_A \zeta_A - h_A)\overline{Y}_A + m_B \overline{Y}_B = - I_f \tag{6-4-1}$$

$$(\rho_B \zeta_B - h_B)\overline{Y}_B + m_A \overline{Y}_A = S_f . \tag{6-4-2}$$

Differentiating (6-4-1) and (6-4-2) with respect to $I_f = S_f$, we get

$$(\rho_A \zeta_A - h_A)\frac{\partial \overline{Y}_A}{\partial I_f} + m_B \frac{\partial \overline{Y}_B}{\partial S_f} = -1$$

$$(\rho_B \zeta_B - h_B)\frac{\partial \overline{Y}_B}{\partial S_f} + m_A \frac{\partial \overline{Y}_A}{\partial I_f} = 1 .$$

Therefore, on the assumption that $(\rho_A \zeta_A - h_A)(\rho_B \zeta_B - h_B) - m_A m_B$ is not zero, we can derive

$$\eta_A \equiv \frac{\partial \overline{Y}_A}{\partial I_f} = \frac{-(\rho_B\zeta_B - h_B + m_B)}{(\rho_A\zeta_A - h_A)(\rho_B\zeta_B - h_B) - m_A m_B} = \frac{-(\rho_B\zeta_B - s_B)}{(\rho_A\zeta_A - s_A - m_A)(\rho_B\zeta_B - s_B - m_B) - m_A m_B}, \tag{6-4-3}$$

$$\eta_B \equiv \frac{\partial \overline{Y}_B}{\partial S_f} = \frac{\rho_A\zeta_A - h_A + m_A}{(\rho_A\zeta_A - h_A)(\rho_B\zeta_B - h_B) - m_A m_B} = \frac{-(s_A - \rho_A\zeta_A)}{(\rho_A\zeta_A - s_A - m_A)(\rho_B\zeta_B - s_B - m_B) - m_A m_B}. \tag{6-4-4}$$

where $s_A = MPS_A = 1 - c_A$ and $s_B = MPS_B = 1 - c_B$.[10] These are what we aim at. (6-4-3) exactly specifies a dynamic marginal effect of capital exports on the trend output of the advanced economy, or the marginal productivity of capital exports. If the constancy of the ratio of the capital exports to the trend output is given, (6-4-3) is a measure of the extent to which the trend growth rate of the advanced economy is

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[10] This assumes that consumer goods are composed of those produced domestically and those imported from abroad.

affected by them. If η_A is positive, this implies that the net national output increases compared with that output which is without any capital exports. It corresponds to that which in the static analysis could be called a capital export multiplier. National income in the present context is smaller than NNP by a positive amount of capital exports. A positive η_A means that national income increases by a smaller amount than NNP. (6-4-3) also indicates that the marginal productivity of capital exports increases in accord with the <u>cet</u>. <u>par</u>. increase of the capital productivity, the saving ratio, the import ratio and the <u>cet</u>. <u>par</u>. decrease of the potential growth rate of the advanced economy.

(6-4-4) measures, in turn, the dynamic marginal effect of capital imports on the equilibrium level of the output of the underdeveloped economy, or the marginal productivity of capital imports. If η_B is positive, the NNP can increase in relation to that before the capital inflow. One of the explanations of this is the positive function of foreign saving to fertilize latent non-capital productive factors. This enables a super-normal growth rate to be achieved for a period, according to R. F. Harrod.[11] In the case of Greece, additional savings out of aid-induced increases in GNP financed a higher proportion of

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[11] R. F. Harrod, 'Desirable International Capital Movements in Relation to Growth of Borrowers and Lenders and Growth of Markets,' <u>International Trade and Theory in a Developing World</u>, 1963, Proceedings of a Conference held by the I.S.A., St. Martin's, Ch. 5, p. 119.

additional investments than the aid itself.[12] η_B corresponds to what in the static analysis could be called a capital import multiplier as set forth by W. Guth.[13]

The condition for one country to benefit by capital movements cannot be determined apart from the situation of the other country. Let us seek the conditions which bring mutually beneficial results. In order for both η_A and η_B to be positive, the denominators and the numerators of (6-4-3) and (6-4-4) must be of the same sign. On close examination, they reduce to

$$I_B/Y_B = \rho_B\zeta_B > s_B + m_B \tag{a}$$

and

$$s_A > \rho_A\zeta_A = I_A/Y_A . \tag{b}$$

(a) and (b) together constitute the necessary and sufficient conditions for the marginal ratio of the capital exports to the output of the advanced economy and for the marginal ratio of the capital imports to the output of the underdeveloped country to be positive. (a) implies that the ex-ante investment ratio must be larger than the sum of the ex-ante saving ratio and the import ratio. In an ex-post context the

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[12] H. B. Chenery, 'Foreign Assistance and Economic Development,' <u>Capital Movements and Economic Development</u>, Proceedings of a Conference held by the I.S.A., 1967, St. Martin's Press, Ch. 7, p. 290.

[13] <u>op</u>. <u>cit</u>. pp. 116-118.

export ratio fills the gap between the combined saving-plus-import ratio and the investment ratio. (b) implies that either the capital-output ratio or the growth rate of the trend output must be so small that the ex-ante investment ratio lies below the ex-ante saving ratio. (b) depicts a typical situation of the advanced economy in that ex-ante saving is excessive relative to ex-ante investment.

The following example is constructed for the purpose of showing the possibility of both countries' benefiting by capital movements. The figures below take (a) and (b) into due consideration.

$\rho_B = 7$	$\rho_A = 10$	(n.b. $\rho_A > \rho_B$)
$\zeta_B = 0.08$	$\zeta_A = 0.04$	(n.b. $\zeta_B > \zeta_A$)
$c_B = 0.7$	$c_A = 0.5$	
$m_B = 0.2$	$m_A = 0.2$	
$s_B = 0.3$	$s_A = 0.5$	

$$\rho_B\zeta_B = 7 \times 0.08 = 0.56 > 0.5 = 0.2 + 0.3 = s_B + m_B$$

$$s_A = 0.5 > 0.4 = 10 \times 0.04 = \rho_A\zeta_A$$

$$\eta_A = \frac{-(0.56 - 0.3)}{-0.048} = 5.42$$

$$\eta_B = \frac{-(0.5 - 0.4)}{-0.048} = 2.08$$

Notice that here we have taken care of the conspicuous fact of the

postwar economic growths: the inverse correlation between the growth rate and the incremental capital-output ratio.[14]

Let us next attempt empirical investigations by means of the data used in the last section. In view of probable wide margins of error involved in using such data, due allowances need to be made for the results:

Empirical Example 1: the U.S.A. (the country A) and Argentina (the country B)

$$\eta_A = 0.030 \;/\; 0.0258 = 1.16$$

$$\eta_B = -0.06 \;/\; 0.0258 = -2.30$$

$$\rho_B\zeta_B = 10.67 \quad 0.03 = 0.32 \nmid 0.75 = 0.36 + 0.39 = s_B + m_B$$

$$s_A = 0.32 > 0.252 = 6.3 \times 0.04 = \rho_A\zeta_A .$$

In the case of the U.S.A. and Argentina, the condition (a) does not hold, although (b) does.

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[14] S. Patel, 'A Note on the Incremental Capital Output Ratio and Rates of Economic Growth in the Developing Countries,' Kyklos, Vol. XXI, 1968, pp. 147-150; M. Shinohara, 'International Comparison of Postwar Capital Coefficients and Growth Rates,' Table 6, Growth of the Japanese Economy, ed. by I. Nakayama, Tokyo, 1960, p. 13; H. Leibenstein, 'Incremental Capital-Output Ratios and Growth Rates in the Short Run,' The Review of Economics and Statistics, Vol. XLVIII, 1966, pp. 20-27. Leibenstein says that the inverse relation between growth rates and ICORs holds for the time series and for cross-sectional data.

<u>Empirical Example 2</u>: the U.K. (the country A') and Argentina (the country B')

$$\eta_{A'} = 0.030 \,/\, 0.0125 = 2.4$$

$$\eta_{B'} = -0.020 \,/\, 0.0125 = -1.6$$

$$\rho_{B'}\zeta_{B'} = 10.67 \times 0.03 = 0.32 \ngtr 0.75 = 0.36 + 0.39 = s_B + m_B$$

$$s_A = 0.26 > 0.128 = 6.4 \times 0.02 = \rho_{A'}\zeta_{A'}$$

In this case, too, the condition (a) fails to be satisfied, although (b) is satisfied.

CHAPTER VII

LINEAR PROGRAMMINGS FOR NARROWING THE GAP BETWEEN AN ADVANCED AND AN UNDERDEVELOPED ECONOMY

Section 1

Introductory Remarks

Optimizing models so far have all been discussed from respective domestic standpoints. Chapter III has dealt with programmings in the interest of an underdeveloped economy. Chapter V has in turn handled optimizing models for an advanced economy. However, these single-country models have left important elements out of the picture — in the world economy with increasing dynamic interdependence. It is necessary, and also of theoretical interest, to build programming models from a global viewpoint with explicit reference to international capital movements.

One of the objectives for this programming might be to maximize world income subject to certain constraints. It, however, leaves much remoteness from practicality. A more practical and realistic approach would be to begin by focusing our attention to the grim truth of widening income gaps between the underdeveloped 'have-not' and the advanced 'have' country. The first problem of this chapter shall be to find such an optimal set of international capital movements and respective national incomes as would bring about a maximum possible homogenization of the unequal national incomes.

An alternative to the first approach would begin by turning our

attention to the standard of living, the first approximate index of which is a per capita real income. The corresponding programming must be reducible to conditional minimization of the gap of per capita real incomes between the advanced and the underdeveloped country. In other words, the second problem of this chapter shall consist in finding such an optimal set of international capital flows as would accomplish a maximum possible homogenization of standards of living.[1] And herein lies a way to make a realistic contribution to further international amity and peace.

Section 2

Linear Programmings for Narrowing the Income Gap between an Advanced and an Underdeveloped Economy

This section aims at building a linear programming model to find that optimal set of capital exports (or imports) and respective national incomes (NI's) which would bring about a least feasible difference of the NI of the advanced economy from the NI of the underdeveloped economy. For this purpose we first adopt or redefine the following notations:

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[1]This kind of gap-narrowing assistance programs is also suggested, though not elaborated, by John C. H. Fei and Douglas S. Paauw, 'Foreign Assistance and Self-Help: A Reappraisal of Development Finance,' The Review of Economics and Statistics. Vol. XLVII, 1965, p. 266.

$K_X(K_M)$: column vector of the flow of capital exports (capital imports)[2]

$y_A(y_B)$: column vector of NI of the advanced economy (the underdeveloped economy)

$\bar{c}_A(\bar{c}_B)$: column vector of autonomous consumption of the advanced economy (the underdeveloped economy)

$c_A(c_B)$; diagonal matrix of marginal propensity to consume of the advanced economy (the underdeveloped economy)

$m_A(m_B)$: diagonal matrix of marginal propensity to import of the advanced economy (the underdeveloped economy)

$J_A = [\rho_{ij}]A \quad (J_B = [\rho_{ij}]B)$: capital coefficient matrix of the advanced economy (the underdeveloped economy) $(i, j = n)$

E: unit row vector.

NIs of both countries are defined by

$$E\, y_A = E\, \bar{c}_A + E\, c_A\, y_A + E\, J_A\, \dot{y}_A + E\, \bar{m}_B + E\, m_B\, y_B - (E\, m_A + E\, m_A\, y_A) - E\, K_X \qquad (7\text{-}2\text{-}1)$$

and

$$E\, y_B = E\, \bar{c}_B + E\, c_B\, y_B + E\, J_B\, \dot{y}_B + E\, \bar{m}_A + E\, m_A\, y_A - (E\, m_B + E\, m_B\, y_B) + E\, K_M \qquad (7\text{-}2\text{-}2)$$

The NI of the advanced economy noticeably exceeds that of its partner. In view of the equality of the total capital exports to the total capital imports, $E\, K_X = E\, K_M$, the absolute positive gap of NIs

[2] K_X and K_M correspond to I_f and S_f in the previous chapter.

becomes

$$E\,y_A - E\,y_B = E\,\bar{c}_A - E\,\bar{c}_B + E\,c_A\,y_A - E\,c_B\,y_B + E\,J_A\,\dot{y}_A$$

$$- E\,J_B\,\dot{y}_B + 2\,E\,\bar{m}_B - 2\,E\,\bar{m}_A + 2\,E\,m_B\,y_B$$

$$- 2\,E\,m_A\,y_A - 2\,E\,K_M\,,$$

from which those terms to be treated as independent of y_A and y_B are eliminated with the following objective function:

$$E\,J_A\,\dot{y}_A - E\,J_B\,\dot{y}_B + E(c_A - 2\,m_A)\,y_A - E(c_B - 2\,m_B)\,y_B - 2\,E\,K_M\,. \tag{7-2-3}$$

To minimize the positive gap of the NIs is tantamount to maximization of the negative gap of the NIs, i.e. $E\,y_B - E\,y_A$. Therefore, instead of minimizing (7-2-3), it is alternatively possible to maximize (7-2-3) with every sign reversed. This alternative objective function takes the form

$$-E\,J_A\,\dot{y}_A + E\,J_B\,\dot{y}_B - E(c_A - 2\,m_A)\,y_A + E(c_B - 2\,m_B)\,y_B + 2\,E\,K_M\,. \tag{7-2-4}$$

Among the constraints to be imposed upon the present maximand (7-2-4) would first of all be considered the condition that total labor demand cannot exceed total labor supply. Denoting the available limits of labor supply of the advanced and the underdeveloped economy by L_A and L_B in vector form, respectively, we can specify the first

constraint on y_A with (5-1-4) in view as

$$\bar{n}_A + n_A y_A \leq .96 L_A , \tag{7-2-5}$$

where $\bar{n}_A$ is the column vector of autonomous employments and n_A is the diagonal matrix of coefficients attached to y_A. (7-2-5) condenses the nature of employment problems common more or less to advanced economies. It leaves, however, something unnoticed to postulate for the underdeveloped economy the same kind of relationship

$$\bar{n}_B + n_B y_B \leq L_B .$$

For the labor demand of the underdeveloped economy falls into long-run and short-run factors. The underdeveloped economy is typically characterized by structural underemployment which can best be remedied by capital accumulation, as has been stated in Section 3, Chapter III. Therefore we need to introduce the total capital stock, approximated by the imported real capital K_M as in Section 2, Chapter III, into the labor demand function. We thus reach the following revised first constraint

$$\bar{n}_B + n_B y_B + \nu K_M \leq L_B , \tag{7-2-6}$$

where ν is a diagonal matrix of the marginal responses of labor demand to capital stock.

We, therefore, can formulate linear programming for filling the gap of NI between the advanced and the underdeveloped economy:

Primal I: Maximize the negative NI-gap (that is tantamount to minimizing the NI-gap) as a function of $\dot{y}_A, \dot{y}_B, y_A, y_B$ and K_M

$$[-E\,J_A,\ E\,J_B,\ -E(c_A - 2m_A),\ E(c_B - 2m_B),\ 2\,E]\ \{\dot{y}_A\ \dot{y}_B\ y_A\ y_B\ K_M\}$$

subject to the labor-supply constraints

$$\begin{bmatrix} 0 & & & & 0 \\ & 0 & & & \\ & & n_A & & \\ & & & n_B & \nu \\ 0 & & & & 0 \end{bmatrix} \begin{bmatrix} \dot{y}_A \\ \dot{y}_B \\ y_A \\ y_B \\ K_M \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ .96L_A - \bar{n}_A \\ L_B - \bar{n}_B \\ 0 \end{bmatrix}$$

and $\dot{y}_A, \dot{y}_B, y_A, y_B, K_M \geq 0$; [3]

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[3] From a more practical viewpoint the primal can take the following format.

Primal I: Maximize the negative NI-gap (that is tantamount to minimizing the NI-gap) as a function of $y_A(t-1)$, $y_B(t-1)$, $y_A(t)$, $y_B(t)$ and K_M

$$[-EJ_A, EJ_B, -E(c_A - 2m_A), E(c_B - 2m_B), 2E]\{y_A(t-1), y_B(t-1), y_A(t), y_B(t), K_M\}$$

subject to the labor-supply constraints

$$\begin{bmatrix} 0 & & & & 0 \\ & 0 & & & \\ & & n_A & & \\ & & & n_B & \\ 0 & & & & 0 \end{bmatrix} \begin{bmatrix} y_A(t-1) \\ y_B(t-1) \\ y_A(t) \\ y_B(t) \\ K_M \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ .96L_A - \bar{n}_A \\ L_B - \bar{n}_B \\ 0 \end{bmatrix}$$

and $y_A(t-1)$, $y_B(t-1)$, $y_A(t)$, $y_B(t)$, $K_M \geq 0$. (t = time)

The Dual I can be rewritten likewise. The same kind of modification can be made of the subsequent programmings.

Dual I: Minimize the objective function

$$[0 \quad 0 \quad .96L_A - \bar{n}_A \quad L_B - \bar{n}_B \quad 0]\ \{z\}$$

subject to

$$\begin{bmatrix} 0 & & & & 0 \\ & 0 & & & \\ & & n_A & & \\ & & & n_B & \\ 0 & & & & 0 \end{bmatrix}' \{z\} \geq \begin{Bmatrix} -E\ J_A \\ E\ J_B \\ -E\ (c_A - 2m_A) \\ E\ (c_B - 2m_B) \\ 2\ E \end{Bmatrix}$$

and $z \geq 0$, where z is the column vector of shadow prices as before.

Amplification of the above simple programming can be done in various ways; for instance, by introducing capital exports as an explicit variable influencing the labor demand of the advanced economy, or by considering explicitly the fact that there is no need to use all current capital equipment, imported or domestic or both, to the full in one or both economies, or by taking into direct account the requirement of rapid growth for the developing economy and the desideratum of stable growth for the advanced economy, or by focusing on balance-of-payments constraints, or by their proper combinations, all depending upon particular situations preconceived. Exact reformulation of the above programming can be carried out by means of the method used in Chapter III and IV, so that the author may well be justified in giving just one example of reformulation along one of the ways suggested: addition of the requirement of rapid growth for the country B and the desideratum of stable growth for the country A.

We suppose that an imaginary world government were powerful and experienced enough to know how to narrow the NI-gap between the 'have' and the 'have-not' country (A and B) subject not only to the labor-supply constraints but also to the requirement of rapid growth on the side of the developing economy and that of stable growth on the part of the developed economy. The required goal of rapid growth for the country B can be specified sectorwise as

$$\dot{y}_{B,i}/y_{B,i} = \max \{\dot{y}_{B,i}/y_{B,i}\} = \xi_{B,i} \text{ (cf. (3-2-2))}$$

or, in matrix form,

$$\dot{y}_B = \xi_B \, y_B \, , \tag{7-2-7}$$

where ξ_B is a diagonal matrix of the required sectoral growth rates predetermined, say, by policy makers of the imaginary world government. The target of the stable growth of the country A is definable as the condition of ex-ante equality of the investment-side and the saving-side in the open economy, i.e.

$$J_A \, \dot{y}_A - H_A \, y_A + m_B \, y_B = d_A \, , \tag{7-2-8}$$

where

$$H_A = [h_{A,i}] = [1 - c_{A,i} - m_{A,i}] \text{ (a diagonal matrix)}$$

and

$$d_A = \overline{m}_A - \overline{m}_B + K_X \, .$$

A more meaningful set of constraints consists now of (7-2-7) and (7-2-8) as well as (7-2-5) and (7-2-6), subject to all of which the objective function (7-2-4) is maximized as follows:

Primal II: Maximize the negative NI-gap (that is tantamount to minimizing the NI-gap) as a function of $\dot{y}_A$, y_A, y_B and K_M

$$[-E\,J_A \quad -E\,(c_A - 2m_A) \quad E(J_B\,\xi_B + \bar{c}_B - 2m_B) \quad 2\,E]\,\{\dot{y}_A \; y_A \; y_B \; K_M\} \tag{7-2-9}$$

$$\begin{bmatrix} J_A & -H_A & m_B & 0 \\ 0 & n_A & 0 & 0 \\ 0 & 0 & n_B & \nu \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y}_A \\ y_A \\ y_B \\ K_M \end{bmatrix} \leq \begin{bmatrix} d_A \\ .96L_A - \bar{n}_A \\ L_B - \bar{n}_B \\ 0 \end{bmatrix}$$

and $\dot{y}_A, y_A, y_B, K_M \geq 0$:

Dual II: Minimize the objective function

$$[d_A \quad .96L_A - \bar{n}_A \quad L_B - \bar{n}_B \quad 0]\,\{z\}$$

subject to

$$\begin{bmatrix} J_A & -H_A & m_B & 0 \\ 0 & n_A & 0 & 0 \\ 0 & 0 & n_B & \nu \\ 0 & 0 & 0 & 0 \end{bmatrix}' \{z\} \geq \begin{Bmatrix} -E\,J_A \\ -E\,(c_A - 2m_A) \\ E\,(J_B\xi_B - \bar{c}_B - 2m_B) \\ 2\,E \end{Bmatrix}$$

and $z \geq 0$.

The solution of the above sets of programs, with or without introduction of some artificial variables, offers that minimum positive NI-gap between the two countries which is feasible under the labor supply constraint with or without the respective domestic growth targets. In other words, the optimal set of capital exports of the advanced economy and capital imports of the underdeveloped economy can thus be brought to light, while diminishing the income gap to the least possible degree subject to the specified constraints. And these can be useful references in a global policy making.

Section 3

Linear Programmings for Narrowing the Per-Capita-Income Gap between an Advanced and an Underdeveloped Economy

The present section is designed to construct programming models for the purpose of narrowing the standard-of-living gap between an advanced and an underdeveloped economy. The standard of living is best approximated by real per capita income, which is in fact an accepted

criterion for classifying developed and developing countries.[4]

Let the size of population of the advanced and the underdeveloped economy be represented by P_A and P_B respectively, and we have straightforward definitions of the respective per capita real incomes:

$$y_A^{p.c.} = \frac{y_A}{P_A} \quad \text{and} \quad y_B^{p.c.} = \frac{y_B}{P_B}, \tag{7-3-1}$$

where $y_A^{p.c.}$ and $y_B^{p.c.}$ are column vectors of the respective per capita real incomes. The problem facing an imaginary world government, which is now more welfare-conscious, changes to one of finding that optimal set of per capita real capital exports and per capita real capital imports as well as per capita real incomes of the respective countries which would bring about a minimum possible per capita real income gap between the advanced and the underdeveloped country. The objective function, the negative gap of per capita real income, need correspondingly be transformed into

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[4] The real gross domestic product per capita is as follows:

Developed countries (a)	1,725	(1965)
Developing countries (b)	157	(1965)

Dollars at 1960 prices

(a) North America, Western Europe, Japan, Australia and New Zealand.

(b) Other countries excluding centrally planned economies.

Source: U.N., World Economic Survey, 1967, N. Y., 1968, p. 19, Table 3.

$$-E\ J_A\ \dot{y}_A^{p.c.} + E\ J_B\ \dot{y}_B^{p.c.} - E(c_A - 2m_A)y_A^{p.c.}$$

$$+ E(c_B - 2m_B)y_B^{p.c.} + E\ K_X^{p.c.} + E\ K_M^{p.c.}, \qquad (7\text{-}3\text{-}2)$$

where $K_X^{p.c.}$ and $K_M^{p.c.}$ are column vectors of the sectoral per capita capital exports of the advanced economy and of the sectoral per capita capital imports of the underdeveloped economy. The maximand (7-3-2) is under the control of considerations of the policy makers who intend to forestall excessive per capita capital movements as well as excessive per capita incomes relative to the availability of labor in the respective countries. The programming is now ready to be formulated:

Primal I: Maximize the negative gap of per capita real income (that is tantamount to minimizing the standard-of-living gap) between the advanced and the underdeveloped country

$$[-E\ J_A \quad E\ J_B - E\ (c_A - 2m_A) \quad E\ (c_B - 2m_B) \quad E \quad E]$$

$$\{\dot{y}_A^{p.c.} \quad \dot{y}_B^{p.c.} \quad y_A^{p.c.} \quad y_B^{p.c.} \quad K_X^{p.c.} \quad K_M^{p.c.}\}$$

subject to the labor supply constraint in terms of population

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & n_A & 0 & 0 & 0 \\ 0 & 0 & 0 & n_B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y}_A^{p.c.} \\ \dot{y}_B^{p.c.} \\ y_A^{p.c.} \\ y_B^{p.c.} \\ K_X^{p.c.} \\ K_M^{p.c.} \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ .96L_A^{p.c.} - \bar{n}_A^{p.c.} \\ L_B^{p.c.} - \bar{n}_B^{p.c.} \\ 0 \\ 0 \end{bmatrix}$$

and $\dot{y}_A^{p.c.}$, $\dot{y}_B^{p.c.}$, $y_A^{p.c.}$, $y_B^{p.c.}$, $K_X^{p.c.}$, $K_M^{p.c.} \geq 0$,[5]

where $K_X^{p.c.} = \dfrac{K_X}{P_A}$, $K_M^{p.c.} = \dfrac{K_M}{P_B}$, $L_A^{p.c.} = \dfrac{L_A}{P_A}$, $L_B^{p.c.} = \dfrac{L_B}{P_B}$, $\bar{n}_A^{p.c.} = \dfrac{\bar{n}_A}{P_A}$

and $\bar{n}_B^{p.c.} = \dfrac{\bar{n}_B}{P_B}$;

Dual I: Minimize the objective function

$$[0 \quad 0 \quad .96L_A^{p.c.} - \bar{n}_A^{p.c.} \quad L_B^{p.c.} - \bar{n}_M^{p.c.} \quad 0 \quad 0] \, \{z\}$$

subject to

[5]The non-negativity conditions $\dot{y}_A^{p.c.}$, $\dot{y}_B^{p.c.} \geq 0$ imply that the growth of population lags behind, or is at most equal to, that of per capita real income in the respective countries.

$$\begin{bmatrix} 0 & \cdots & & & & 0 \\ \vdots & 0 & & & & \vdots \\ \vdots & & n_A & & & \vdots \\ \vdots & & & n_B & 0 & \nu \\ \vdots & & & & 0 & 0 \\ 0 & \cdots & & & & 0 \end{bmatrix}' \{z\} \geq \begin{Bmatrix} -E\ J_A \\ E\ J_B \\ -E(c_A - 2m_A) \\ E(c_B - 2m_B) \\ E \\ E \end{Bmatrix}$$

and $z \geq 0$.

When it is possible to take into additional account the goal of rapid growth for the underdeveloped economy and the target of stable growth for the advanced economy, the objective function and the constraint must undergo corresponding modifications, as presented below:

Primal II: Maximize the negative gap of per capita real income (that is, minimize the standard-of-living gap) between the two countries

$$[-E\ J_A - E\ (c_A - 2m_A) \quad E(J_B \xi_B + \bar{c}_B - 2m_B) \quad 2\ E]$$

$$\{\dot{y}_A^{p.c.} \quad y_A^{p.c.} \quad y_B^{p.c.} \quad K_M^{p.c.}\}\ , \qquad (7\text{-}3\text{-}3)$$

subject to the labor-demand-supply condition, the rapid-growth target for the country B and the stable-growth goal for the country A

$$\begin{bmatrix} J_A & -H_A & m_B & 0 \\ 0 & n_A & 0 & 0 \\ 0 & 0 & n_B & \nu \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y}_A^{p.c.} \\ y_A^{p.c.} \\ y_B^{p.c.} \\ K_M^{p.c.} \end{bmatrix} \leq \begin{bmatrix} d_A^{p.c.} \\ .96L_A^{p.c.} - \bar{n}_A^{p.c.} \\ L_B^{p.c.} - \bar{n}_B^{p.c.} \\ 0 \end{bmatrix}$$

and $\dot{y}_A^{p.c.}$, $\dot{y}_A^{p.c.}$, $\dot{y}_B^{p.c.}$, $K_M^{p.c.} \geq 0$,

where $$d_A^{p.c.} = \frac{\bar{m}_A}{P_A} - \frac{\bar{m}_B}{P_A} + \frac{K_X}{P_A} ;$$

Dual II: Minimize the objective function

$$[d_A^{p.c.} \quad .96L_A^{p.c.} - \bar{n}_A^{p.c.} \quad L_B^{p.c.} - \bar{n}_B^{p.c.} \quad 0] \{z\}$$

subject to

$$\begin{bmatrix} J_A & -H_A & m_B & 0 \\ 0 & n_A & 0 & 0 \\ 0 & 0 & n_B & \nu \\ 0 & 0 & 0 & 0 \end{bmatrix}' \{z\} \geq \begin{Bmatrix} -E\ J_A \\ -E\ (c_A - 2m_A) \\ E\ (J_B\ \xi_B + \bar{c}_B - 2m_B) \\ 0 \end{Bmatrix}$$

and $z \geq 0$.

The solution to be derived presents the narrowest possible gap of living standards between the advanced and the underdeveloped country on the condition that the labor demand-supply relation with or without the

respective domestic growth targets is guaranteed. The optimal per capita capital exports and the optimal per capita capital imports as well as the respective per capita incomes should be compared with corresponding existing figures to determine relative deficits or surpluses.

Before concluding this chapter it may be added that all of the above programs incorporate the requirement of universal full employment, though not explicitly. The global full-employment target requires that a strict equality exist in each of the above labor deman-supply relations instead of the less-than-or-equal-to sign. The target of universal full employment can enter programming explicitly for the advanced economy as

$$n_A \, y_A = .96L_A - n_A \quad \text{(Section 2)}$$

or,

$$n_A \, y_A^{p.c.} = .96L_A^{p.c.} - \overline{n}_A^{p.c.} \quad \text{(Section 3)}$$

and for the underdeveloped economy as

$$n_B \, y + \nu \, K_M = L_B - \overline{n}_B \quad \text{(Section 2)}$$

or,

$$n_B \, y_B^{p.c.} + \nu \, K_M^{p.c.} = L_B^{p.c.} - \overline{n}_B^{p.c.} \quad \text{(Section 3)} .$$

The fact that no slack variable is needed necessitates the introduction

of some set of artificial variables in order to maintain a ready-made initial best feasible solution. Except for this, the corresponding programs are of the same kind and can be solved in the same way.

Replacement of the above continuous programs by corresponding discrete ones will meet more practical purposes such as prediction.

Mention may also be made of an extension of the two-country programming to that of multi-advanced and multi-underdeveloped world economy. This extension begins by regarding anew each element of the vectors or the matrices used as consisting of each domestic element or matrix. Each notation being thus redefined, each definitional equation, each objective function, each constraint inequality or equality, and each growth target assume the same appearance in the programming, though it deals with the groups, advanced and underdeveloped, each group being of many disaggregated economies.[6]

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[6] On the assumption that there are N_A advanced and N_B underdeveloped economies, redefinition shall be exemplified for $y_A, K_M^{p.c.}$, J_A and n_B. $y_A = \{y_{A-1}, y_{A-2}, \cdots, y_{A-i}, \cdots, y_{A-N_A}\}$, where $y_{A-i} = \{y_{A-i-1}, y_{A-i-2}, \cdots\}$ and e.g. y_{A-i-2} implies the income of the 2nd sector of A-i^{th} advanced economy. $K_M^{p.c.} = \{K_{M:B-1}^{p.c.}, K_{M:B-2}^{p.c.}, \cdots, K_{M:B-i}^{p.c.}, \cdots, K_{M:B-N_B}^{p.c.}\}$, where $K_{M:B-i}^{p.c.}$ is the column vector of sectoral per capita imports of the B-i^{th} underdeveloped economy. $J_A = [J_{A-i,A-J}]$, where $J_{A-i,A-j}$ is the sectoral capital coefficient matrix between the sectors of the A-i th advanced and those of the A-j^{th} advanced economy. n_B is the diagonal matrix of the order $N_B \times N_B$ with each element being of a domestic diagonal matrix of the respective underdeveloped economies. The other notations being thus redefined, the definitional equations, the objective functions, the constraints and the other specifications are to be interpreted in the same amplified context.

The knowledge concerning the pivoting in programming reveals that any autonomous increase of capital exports, resulting in the larger constant term of the steady-advance condition of the developed economy, brings about the further improvement of the optimal value of the objective function. This applies to Primal and Dual II of Section 2 and 3. In other words, the gaps of the national income and the living standard can further be narrowed by autonomous increase of the capital exports, while the stable growth of the advanced and the rapid development of the underdeveloped economy being simultaneously guaranteed.

CHAPTER VIII

THE BALANCE OF PAYMENTS ON CURRENT ACCOUNT OF AN ADVANCED AND AN UNDERDEVELOPED ECONOMY

Section 1

Introductory Specifications

The present chapter is designed as a supplement to the analysis of real aspects by which the previous chapters, except Chapter I and the Appendix, Chapter II, may be characterized. We will first bring out dynamic behaviors of the income account of each country and derive a required amount of financial unilateral capital movements in the context of a world economy composed of a growing advanced and a developing underdeveloped economy. This requires us to analyze the commodity terms of trade and the balance of payments on current account of the respective economies, after due specification of what is meant by financial capital exports and imports here. Then we will proceed to analyze the marginal effects of exports and capital movements on the balance of payments of each country.

Autonomous financial capital exports, thus far loosely defined, are here to be considered as a monetary item to be entered on the debit side of the balance of payments on current account. They can be regarded as more or less synonymous with donations (gifts) to foreign countries. In this sense they are usually of an unrequited nature, being independent of pecuniary motives; this would justify the simplifying assumption for us to neglect interest, dividend or other factor

payments. The economic function of the autonomous capital movements which does not appear in accounting statements is that of real factor of production; this explains why we preferred the expression 'autonomous investment abroad' in Chapter VI. The balance of payments on current account (income account) of the advanced economy now consists of the accounts of the donations as well as goods and services.

The other side of the story concerns autonomous capital imports. The capital account which records monetary transactions based on pecuniary motives is not assumed to include the present item of autonomous capital imports. Their economic role is symbolized by the expression 'saving from abroad,' an addition to the scanty source of domestic saving, as used in Chapter VI. In brief, with interest, dividend or other factor payments assumed away, as above, the income account of the underdeveloped economy here is made up of the accounts of gifts along with visible and invisible trade.

Section 2

The Balance of Payments on Current Account of the Respective Economies

Explicit presentation of the income account and its dynamic behavior in the advanced and the underdeveloped economy is in order. Introducing the notations,

$p_{x_{A(B)}}$: index number of the export prices of the advanced (the underdeveloped) economy

$p_{m_{A(B)}}$: index number of the import prices of the advanced (the underdeveloped) economy

p : index number of the financial unilateral capital movements,

we can define the balance of payments on current account of the advanced and the underdeveloped economy, denoted respectively by B_A and B_B, as

$$B_A = p_{x_A} X_A - p_{m_A} M_A - p\, I_f \tag{8-2-1}$$

$$B_B = p_{x_B} X_B - p_{m_B} M_B + p\, S_f \,. \tag{8-2-2}$$

The foreign exchange rate is assumed to be 1:1 . The overall account of the two-country world is obviously in balance.

In view of the respective import functions connecting $M_A(M_B)$ with $Y_A(Y_B)$ (Section 2, Chapter VI), we combine (8-2-1) and (8-2-2) with the respective national output paths (6-2-12) and (6-2-13) to obtain the dynamic behavior of each income account:

$$\begin{aligned} B_A = & -p\, I_f + p_{m_A}(\phi_A \bar{m}_B - \bar{m}_A) + p_{m_A}(\phi_A m_B \bar{Y}_B - m_A \bar{Y}_A) \\ & + p_{m_A}(\phi_A m_B \beta_1 - m_A \alpha_1)e^{r_1 t} + p_{m_A}(\phi_A m_B \beta_2 - m_A \alpha_2)e^{r_2 t} , \end{aligned} \tag{8-2-3}$$

$$B_B = p\, S_f + p_{m_B}(\phi_B\, \overline{m}_A - \overline{m}_B) + p_{m_B}(\phi_B\, m_A\, \overline{Y}_A - m_B\, \overline{Y}_B)$$

$$+ p_{m_B}(\phi_B\, m_A\, \beta_1 - m_B\, \alpha_1)e^{r_1 t} + p_{m_B}(\phi_B\, m_A\, \beta_2 - m_B\, \alpha_2)e^{r_2 t} \qquad (8\text{-}2\text{-}4)$$

where ϕ_A is the commodity terms of trade for the advanced economy

$$\phi_A = p_{x_A}/p_{m_A} \qquad (8\text{-}2\text{-}5)$$

and ϕ_B is the counterpart for the underdeveloped economy

$$\phi_B = p_{x_B}/p_{m_B} . \qquad (8\text{-}2\text{-}6)$$

We should approach cautiously the tacit assumption that ϕ_A and ϕ_B are invariant through the process of growth or development, although the commodity terms of trade are subject to various changes depending on types of development: ultra-import-biased, import-biased, neutral, export-biased, ultra-export-biased.[1]

With a view to arriving at some meaningful results, let us make the following simplifying but not implausible assumption. The advanced

[1]See, e.g., Gerald M. Meier, International Trade and Development, Harper & Row, N. Y. 1964, Ch. 3. The assumption seems to have typical plausibility in the context of the underdeveloped economy, if we look on it as the same what T. W. Swan calls a dependent economy: T.W. Swan, 'Economic Control in s Dependent Economy,' Economic Record, March 1960, p. 53. A brief analysis of variable terms of trade has been given in the Appendix to Chapter II though. An interest similar to Meier's occupies the mind of Jagdish Bhagwati in his paper, 'Growth, Terms of Trade and Comparative Advantage.' Economia Internationale, Vol. 12, 1959, pp. 381-418.

economy is assumed to grow along the equilibrium part of its output $\overline{Y}_A(t)$ at a smaller time rate than the underdeveloped economy does along its own $\overline{Y}_B(t)$:

$$\dot{\overline{Y}}_A / \overline{Y}_A = \zeta_A < \zeta_B = \dot{\overline{Y}}_B / \overline{Y}_B . \tag{8-2-7}$$

This assumption will prove to play a significant role in specifying the secular behavior of the respective income accounts B_A and B_B. By means of (8-2-7) the income accounts (8-2-3) and (8-2-4) can be rewritten as

$$B_A = -p\, I_f + p_{m_A}(\phi_A \overline{m}_B - \overline{m}_A) + p_{m_A}(\phi_A m_B \beta_1 - m_A \alpha_1)e^{r_1 t}$$
$$+ p_{m_A}(\phi_A m_B \beta_2 - m_A \alpha_2)e^{r_2 t} + p_{m_A}\left[\phi_A m_B \overline{Y}_B(0) - m_A \overline{Y}_A(0)e^{(\zeta_A - \zeta_B)t}\right] e^{\zeta_B t}$$

(8-2-3)'

$$B_B = p\, S_f + p_{m_B}(\phi_B \overline{m}_A - \overline{m}_B) + p_{m_B}(\phi_B m_A \beta_1 - m_B \alpha_1)e^{r_1 t}$$
$$+ p_{m_B}(\phi_B m_A \beta_2 - m_B \alpha_2)e^{r_2 t} + p_{m_B}\left[\phi_B m_A \overline{Y}_A(0)e^{(\zeta_A - \zeta_B)t} - m_B \overline{Y}_B(0)\right] e^{\zeta_B t}$$

(8-2-4)'

Since $B_A(t)$ and ϕ_A are the respective mirror images of $B_B(t)$ and ϕ_B so that results for the country B imply those for the country A upon due transposition, we have only to consider the case of one country, say, the country B .

Table 8-2-1

Long-run Trend of the Balance of Payments on Current Account of the Underdeveloped Economy

Case I: $r_2 > \zeta_B$		Case II: $r_2 < \zeta_B$	
$\phi_B > \omega_B$	$\phi_B < \omega_B$		
$B_B > 0$ $\dot{B}_B > 0$	$B_B > 0$ up to t^*; $B_B < 0$ after t^*; $\dot{B}_B < 0$	$B_B > 0$ up to t^*; $B_B < 0$ after t^*; $\dot{B}_B < 0$	$B_B(0) > 0$ or $\phi_B > \bar{\phi}_B$
$B_B < 0$ up to t^* $B_B > 0$ after t^* $\dot{B}_B > 0$	$B_B < 0$ $\dot{B}_B < 0$	$B_B < 0$ with $\dot{B}_B < 0$ in the very long run	$B_B(0) < 0$ or $\phi_B < \bar{\phi}_B$

N.B.

$$\bar{\phi}_B = \frac{S_f + \bar{m}_B + m_B\bar{Y}_B(0) + m_B\alpha_1 + m_B\alpha_2}{\bar{m}_A + m_A\bar{Y}_A(0) + m_A\beta_1 + m_B\beta_2}, \quad \omega_B = \frac{m_B\alpha_2}{m_A\beta_2}, \quad B_B(t^*) = 0 .$$

The secular behavior of $B_B(t)$ depends on the relation between ζ_B and the larger characteristic root r_2, on the coefficient of $e^{\zeta_B t}$ and $e^{r_2 t}$, and on the initial value of $B_B(0)$. Short-run behavior being ignored, the balance of payments on current account of the underdeveloped economy would in the long run tend to behave as represented in Table 8-2-1.

This illustration shows parametric functions of the commodity

terms of trade in the picture of the current account of the underdeveloped economy. Case I specifies r_2 as the factor determining the secular behavior of the income account. On the one hand where the given terms of trade is so low as to be $\bar{\phi}_B > \phi_B < \omega_B$, Table 8-2-1 shows that the economy would be placed in a state of constant, and growing, import surplus. It would suffer from balance-of-payments difficulties, unless unilateral capital transfers on current account were on the increase, the capital account were being improved so as to keep the overall balance in equilibrium, or some effective commercial policy were carried out to raise the existing terms of trade. On the other hand where the given terms of trade are high enough to exceed both $\bar{\phi}_B$ and ω_B, the economy would enjoy a growing export surplus.

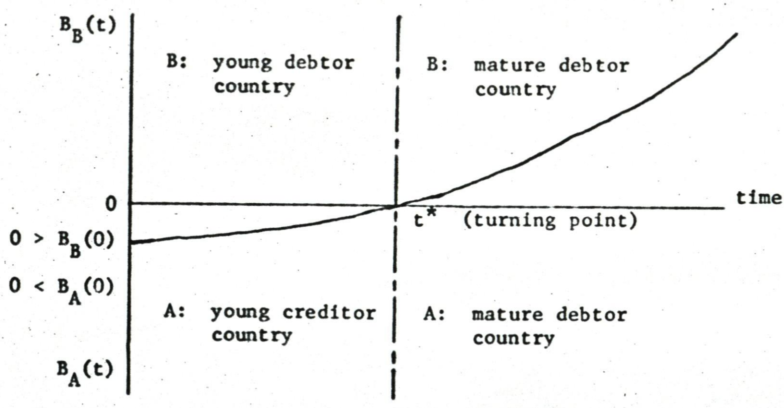
In the intermediate cases, $\bar{\phi}_B > \phi_B > \omega_B$ and $\omega_B > \phi_B > \bar{\phi}_B$, the relation of $\bar{\phi}_B$ to ϕ_B constitutes a key factor determining an upward or a downward divergence of the export surplus. A growing export surplus results when $\bar{\phi}_B$ exceeds ϕ_B; in the case where $\bar{\phi}_B$ is below ϕ_B, the export surplus is on the downgrade. The case $\bar{\phi}_B > \phi_B > \omega_B$ deserves special attention. This can be explained by a historical transition of a young debtor country into a mature debtor country, on the assumption to regard the underdeveloped and the advanced economy respectively as a young debtor and a young creditor country. The first stage up to t^* observes an import surplus $B_B < 0$ with an implied positive capital account due to inflow of financial capital. The second stage after t^* is characterized by an export surplus $B_B > 0$ with an implied negative capital account due to excessive outflow of interest and dividend payments. Viewed from the standpoint of the

partner, country A, a potential creditor country, would in turn witness a process of a young creditor country shifting to a mature creditor country. Its growing deficit on current account in the second stage after t^* tends to be offset by a growing surplus of return flows from past investments recorded on capital account. This simultaneous transition of the income accounts of both countries is summarized in Figure 8-2-1. The familiar argument about this kind of classification of the type of the income account by stages of development has turned out to presuppose the condition $\overline{\phi}_B > \phi_B > \omega_B$ for the assumed world of an underdeveloped debtor and an advanced creditor country.

Figure 8-2-1

The Income Accounts of Country A and B

by Stages of Long-Run Development



Case 2 admits of more difficult interpretations. Whenever the growth rate of the secular equilibrium part of output $\overline{Y}_B(t)$ dominates

the behavior of the general solution of output (6-2-12), the tendency is a continuous deterioration of the balance of payments on current account against the underdeveloped economy irrespective of the initial value of $B_B(t)$, i.e. $B_B(0)$. Since the coefficient of $e^{\zeta_B t}$ is itself a declining function of time (cf. (8-2-4)'), the larger the value ϕ

$$\phi = \frac{m_A\ \overline{Y}_A(0)}{m_B\ \overline{Y}_B(0)}$$

is, the less rapidly such a deterioration of the income account tends to occur. That is, the larger the marginal propensity to import of the advanced economy, the more prolonged will be the period of the balance-of-payments crisis confronting the underdeveloped economy. The further away upwards ϕ_B is from $\overline{\phi}_B$, the further away such a crisis tends to recede.

This analysis can be useful references in predicting a relative influence of an existing terms of trade of the developing country upon a future trend of its income account. Once the actual values of $\overline{\phi}_B$, ω_3 and ζ_B have been estimated, it is possible to determine a relative position of the existing terms of trade ϕ_B and the dominant root r_2 in Table 8-2-1. Based on this determination, the underdeveloped economy as a potential debtor country can calculate the amount of needed future donations from the advanced creditor country. For instance, if the country has proved to be in a position

$$r_2 > \zeta_B \quad \text{and} \quad \omega_B > \phi_B > \overline{\phi}_B ,$$

the required amount of the financial capital imports will amount to $\sum_{t=t^*}^{T} B_B(t)$, where T is a terminal point of some multi-period plan.

A word about policy implications may be added. The above analysis gives a hint for the commercial policy of the underdeveloped economy. What should the country do if the terms of trade are lower than ω_B ? The answer is: it should, if it could, try to lower first the marginal propensity to import m_B through commercial policies such as increase of import taxes, promotion of import substitution, etc. Next it should also try to enhance the value of β_2. However, policy-makers of the developing country need to be cautious enough not to lower m_B to the detriment of rapid development which is the kernel of policy for the developing country. The same kind of cautious attitude is in turn needed for policy planners of the advanced economy; they are warned against exclusively aiming at a positive balance in the income account. An important attitude to take is one of considering commercial policy in conjunction with the overall target of economic growth or development, which applies to any of the economies considered.

Section 3

The Dynamic Effects of Exports on the Balance of Payments on Current Account of the Respective Countries

By using the dynamic exports multiplier developed in Section 3, Chapter VI, we can determine a combined dynamic influence of the

exports upon the balance of payments on current account of each country. In a dynamic setting exports first affect income and then imports, so that the change in income account must be a combination of related factors. We combine (6-3-4) and (8-2-1) in order to obtain the marginal change of the income account of the advanced economy as a result of increase of its exports

$$\frac{\partial B_A}{\partial (p_{x_A} X_A)} = 1 - \frac{m_A}{\phi_A}\frac{\partial Y_A}{\partial X_A} = 1 - \frac{m_A}{\phi_A}\kappa_A . \qquad (8\text{-}3\text{-}1)$$

Capital exports are assumed to be unrelated to exports.[2] Obviously, $\dot{B}_A$ is an increasing function of ϕ_A .

Likewise, we use (6-3-5) and (8-2-2) in combination so as to attain the marginal change of the income account of the underdeveloped economy in consequence of the increase in its exports

$$\frac{\partial B_B}{\partial (p_{x_B} X_B)} = 1 - \frac{m_B}{\phi_B}\frac{\partial Y_B}{\partial X_B} = 1 - \frac{m_B}{\phi_B}\kappa_B . \qquad (8\text{-}3\text{-}2)$$

Since by definition $B_A + B_B = 0$, we get $\frac{\partial B_A}{\partial X_A} = -\frac{\partial B_B}{\partial X_A}$.

On the assumption that $\phi_A = \phi_B = 1$, we can get actual figures of (8-3-1) and (8-3-2) by using the data used in Section 3, Chapter VI.

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[2]This is a simplifying assumption. When capital exports are positively related to exports, the marginal effect of exports on the income account becomes so much larger.

The U.S.A. (the country A):

$$\frac{\partial B_A}{\partial (p_{x_A} X_A)} = 1 - 0.02 \times 11.36 = 0.78, \text{ implying } \frac{\partial B_B}{\partial (p_{x_A} X_A)} = -0.78 .$$

The U.K. (the country A'):

$$\frac{\partial B_{A'}}{\partial (p_{x_{A'}} X_{A'})} = 1 - 0.11 \times 1.62 = 0.822, \text{ implying } \frac{\partial B_B}{\partial (p_{x_{A'}} X_{A'})} = -0.822 .$$

Argentina (the country B):

$$\frac{\partial B_B}{\partial (p_{x_B} X_B)} = 1 - 0.39 \times 2.32 = 0.10, \text{ implying } \frac{\partial B_A}{\partial (p_{x_B} X_B)} = -0.10 .$$

The Argentine economy can improve the balance of payments by the expansion of exports least of all, because it has the highest marginal propensity to import, although its dynamic export multiplier is relatively low (cf. Section 3, Chapter VI). The U.S.A., on the other hand, can increase its income account much more than Argentina, despite the fact its marginal ratio of income to exports is the highest. This is because it has the lowest propensity to import. As for the U.K. the marginal effect of exports on output is lowest of all, and the marginal propensity to import is relatively low (cf. Section 3, Chapter VI). This explains why the increase of imports as a result of increased exports is the smallest. Both the U.S.A. and the U.K. are suggested to increase their respective exports to Argentina if they want to improve their balance of payments position with Argentina. If Argentina finds

itself hard to increase its exports or to decrease its relatively high import ratio, it should also try to improve the terms of trade in which case concessionary attitudes are required on the part of the advanced countries.

Section 4

The Dynamic Effects of Capital Movements upon the Balance of Payments on Current Account of the Respective Countries

The extent to which the outputs of the advanced and the underdeveloped economy are influenced by the capital movements (Section 4, Chapter VI) play here an important role. We first make a simplifying assumption that $p_{x_i} = p_{m_i} = 1 (i = A,B)$. The eventual effect of capital exports on the income account of the advanced economy is derived by combining (6-4-3), (6-4-4) and (8-2-1):

$$\frac{\partial B_A}{\partial I_f} = m_B \frac{\partial Y_B}{\partial S_f} - m_A \frac{\partial Y_A}{\partial I_f} = m_B \eta_B - m_A \eta_A . \qquad (8\text{-}4\text{-}1)$$

Since $B_A + B_B = 0$ (a zero-sum game) by definition, this implies for the underdeveloped economy

$$\frac{\partial B_B}{\partial S_f} = m_A \frac{\partial Y_A}{\partial I_f} - m_B \frac{\partial Y_B}{\partial S_f} = m_A \eta_A - m_B \eta_B , \qquad (8\text{-}4\text{-}2)$$

which is the combined effect of the capital imports on the income

account of the underdeveloped economy. We use now the data used in Section 4, Chapter VI so as to calculate numerical values of (8-4-1) and (8-4-2):

	$\partial B_A / \partial I_f$	$\partial B_B / \partial S_f$
Hypothetical Example	- 0.668	0.668
Empirical Ex. 1	- 1.129	1.129
Empirical Ex. 2	- 0.672	0.672

These examples are based on the simplifying assumption that the coefficients are all constant, so that such changes as import substitution or export substitution are overlooked. They suggest that, in order for the capital exports to improve the balance of payments position for the advanced economy, the capital import multiplier as well as the import ratio of its partner must be sufficiently large. On the other hand, a sufficiently large capital export multiplier and an import ratio are needed in order for the capital import to bring about net improvements in the balance of payments of the underdeveloped economy.

CHAPTER IX

CONCLUSIONS

Section 1

Conclusions of Each Chapter

Chapter 1: The growth rate of domestic equilibrium (potential) output can increase as a result of a rise in capital imports (inflow of saving) and it can decrease as a result of a rise in capital exports (outflow of saving), when only one economy is taken into account. Both capital imports and capital exports can help attain a dynamic internal-external equilibrium.

Chapter 2: An internal-equilibrium criterion of investment allocation can give a useful clue to the determination of needed sectoral capital imports for an underdeveloped economy. A given amount of foreign-aid capital can attain a rapid growth in the underdeveloped economy, if used to procure capital goods for investment purposes under favorable conditions.

Chapter 3: The solutions of models optimizing the net national product, the accumulation of capital and the growth rate of an underdeveloped economy can be useful references in considering its policy for a rapid growth. Additional capital imports result in an improvement of the optimal net national product, the optimal capital accumulation, or the optimal growth rate, so long as non-capital complementary factors and facilities are available in the underdeveloped economy.

Chapter 4: Leads and lags of one sectoral income relative to another are a cause of the cylical growth of an advanced economy. Sufficiently manipulative parameters of its capital exports can result in more moderate oscillations of the national income around its moving equilibrium path, when the other economy stays outside the picture.

Chapter 5: The solutions of programs for the optimal employment and the optimal consumption of an advanced economy can be helpful in studying its policy for a stable growth. The optimal value of the employment or the consumption can be improved through increased capital exports, so long as unemployed productive factors are available.

Chapter 6: In a world consisting of an advanced and an underdeveloped economy both national outputs can increase through increased international capital movements under favorable conditions. The required favorable conditions are first that the investment ratio must exceed the saving ratio plus the import ratio in the underdeveloped economy, and second that the saving ratio must exceed the investment ratio in the advanced economy. National outputs can also increase through increased mutual exports of goods and services under certain favorable circumstances.

Chapter 7: Global programmings for narrowing the national-income or the living-standard gap between an advanced and an underdeveloped economy can produce valuable references in determining relative deficiencies of sectoral capital movements. The international capital movements can help narrow the gap between the two under favorable conditions of population growth.

Chapter 8: The long-run trend study of the balance of payments can be helpful in predicting future needs of capital movements between an advanced and an underdeveloped economy. The autonomous capital exports can improve the balance of payments of the advanced economy if the capital import multiplier as well as the import ratio of its partner are sufficiently large. The autonomous capital imports can exert a further favorable effect on the balance of payments of the underdeveloped economy if the capital export multiplier and the import ratio of its partner are large enough.

Section 2

Concluding Generalizations

We have thus reached a provisional conclusion in each chapter by isolating the complicating aspects of our problem one by one. The next task is to go back on ourselves and allow, as far as we can, for the probable interactions of the aspects among themselves[1] and for related matters which have been overlooked. We have pre-stated the three major problems of our study. The first of these is how the capital imports affect the rapid growth of an underdeveloped economy (Chapters II and III). The second is how the capital exports influence the stable growth of an advanced economy (Chapters IV and V), The third is how

[1] J. M. Keynes, The General Theory of Employment, Interest and Money, Macmillan, London, 1957, p. 295.

far the effects of the international capital movements interact upon the respective open economies of the world (Chapters VI, VII and VIII). In pursuing the first problem we abstract from the aspect of the advanced economy which exports capital, while the second problem is approached in the context which excludes the underdeveloped economy. The third problem is pursued as a partial answer to the problem of this concluding section. These problems must now be discussed in a wider context.

In pursuing the first problem we have taken for granted that the imported capital is used for investment (Chapter II). For we have shown, on the favorable assumption about the absorptive capacity of the underdeveloped economy, that a maximum growth rate of output can be achieved when the imported capital is used to procure capital goods for investment rather than consumer goods. The capacity to absorb highly productive capital goods from an advanced economy presupposes adequate overhead facilities, administrative efficiency, competent entrepreneurs, non-capital complementary factors, trained manpower, mobility of labor and established markets. When these factors do not co-exist sufficiently, the above assumption and the conclusion must become hard to maintain.

Important as these factors are in relation to the absorptive capacity, each of these is considered less important than the real capital in our study. It is the real capital that is regarded as the most scarce factor associated with underdevelopment. A continuous increase of this core factor enhances the upper limit of the capacity output of the underdeveloped economy at an increasing rate (especially

in an early stage of development). This is the background of Chapter III in which the growth rate optimized subject to the capital stock constraint and the foreign trade constraint, is found to be a positive function of capital imports (on the favorable assumption about solvability). The optimal net national product and the optimal capital accumulation are also found to be a positive function of capital imports for the same reason. These optimizing models can be bases of actual plannings. Widely developed simplex algorithms can work out solutions to these programmings. Unlike the classical context of international capital movements or historical interpretations of the 19-th century foreign investments, the present-day role of foreign capital in underdeveloped economies cannot be discussed apart from development planning.

The above optimizing models are based on the multisectoral approach to growth (Chapter II). This retains a definite advantage over the usual macro (one-sector) counterpart, because it enables the policy makers to single out those sectors which are weak in their contributions to overall rapid growth and apply suitable selective long-run policies. It also takes care of Harrod's statement that "growth is the aggregated effect of a great number of individual decisions."

In the absence of the constraints subject to which the growth rate is optimized, we derived the growth rate of output from our basic Harrod-Domar open-economy model (Chapter I). The inflow of saving (capital), autonomous or compensatory, turns out to raise the growth rate of the underdeveloped economy (so long as the other coefficients and factors remain unchanged). This observation is in line with the

orthodox view that foreign investment has a potentially crucial role to play in the growth of underdeveloped areas (R. J. Ball).

The net effects of capital imports on the underdeveloped economy becomes more complicated when the repercussion of the advanced economy comes into the picture (Chapter VI). The marginal ratio of the net national product to the capital imports (the marginal productivity of the capital imports, or the dynamic capital imports mulitplier) becomes positive or negative depending on the combination of the structural coefficients of both countries: the capital coefficients, the growth rate of the trend outputs, and the marginal propensities to save and to import. On the part of the underdeveloped economy, there must exist a certain favorable combination of its endogenous coefficients as well as non-capital cooperative factors and facilities, in order for it to have a net gain. A positive marginal productivity of the capital imports requires that the underdeveloped economy have an ex-ante investment ratio high enough to exceed the sum of the saving and the import ratio on the assumption that a favorable combination of the endogenous coefficients is attained in the advanced economy. Under these conditions, a rapid growth of the developing economy becomes feasible. The higher the marginal productivity of the capital import and the growth rate of output, the earlier will the recipient be able to finance its development out of its own resources and finish its interest and amortization payments, if any.

When compared with the U.S.A., the recipient Argentina has a relatively low growth rate of the trend output and a relatively high capital-output ratio. Its ex-ante investment ratio does not exceed the

sum of the saving and the import ratio, so that the above domestic condition of the underdeveloped economy fails to be satisfied. The U.S.A. (the donor) meets its required domestic condition, for its ex-ante saving ratio exceeds its ex-ante investment ratio. The result is that the capital movements between the two place Argentina in an unfavorable situation. Unless the relatively low growth rate of the trend output were raised or the relatively high import ratio were remedied in Argentina, only the U.S.A. would benefit through capital movements. In terms of the balance of payments (Chapter VIII), however, the U.S.A. is placed in a more unfavorable position. The eventual increase of the U.S. income and imports is larger than the increment of its exports caused solely by the increase of Argentine income and imports in the assumed Argentina-U.S.A. world of the postwar period.

A similar observation holds in the assumed Argentina-U.K. world of the postwar era. The U.K. satisfies its required domestic condition for mutually beneficial capital movements. Argentina fails to meet the required domestic condtion, for its growth rate of the trend output is too low relative to the saving and the import ratio. In consequence, the capital movements between the two place Argentina again in an unfavorable situation in terms of incremental output. The dynamic marginal ratio of the autonomous capital imports to the trend output turns out to be positive only for the U.K. The imports of Argentina, however, increase less than its exports (Chapter VIII). This places it in a better balance-of-payments situation (unless the <u>cet</u>. <u>par</u>. assumption breaks down, say, by import substitution process).

In these two empirical examples the capital-exporting country (the

U.S.A. or the U.K.) gains at the relative cost of the capital importing country (Argentina). On the other hand, the balance-of-payments situation turns for the capital-importing country. If, in addition, the terms of trade assume a favorable value, our analysis shows the process of an interesting simultaneous transition of the assumed young-debtor underdeveloped into the assumed mature-debtor underdeveloped country and of the assumed young-creditor-advanced into the assumed mature-creditor-advanced country. On the assumption that the other country's situation remains unchanged, the capital-importing country can enhance its gain in terms of the incremental net national output, first by lowering its capital-output ratio (through some institutional change leading to a higher labor productivity and a lower capital-labor ratio), second by raising its saving ratio (through some effective taxation or income redistribution) and third by raising its import ratio (more importation of productive goods or factors).

Our second major problem is the effects of capital exports on the stable growth of the advanced economy. Our endogenous multisectoral model of cyclical growth reveals that the behavioral stability of the warranted growth of the national income as the aggregate of the sectoral net incomes depends on a certain combination of endogenous parameters: the sectoral capital coefficients and the sectoral marginal propensities to save and to import. The conditions of explosive, regular and damped fluctuations of the national income turn out to hinge on these endogenous parameters. Under the condition of damped fluctuations, we reached the view that leads or lags of one sectoral income relative to another constitute oscillatory elements of the overall economic growth.

Thus, if capital exports are sufficiently manipulative, they can help stabilize the cyclical movements of the growing economy (Chapter IV). This provisional conclusion must now be subjected to reconsideration in view of the repercussion of the rest of the world. The problem is that this deductive conclusion is not quite relevant to the other country. The policy of counter-cyclical capital exports has the potential danger of exposing the capital-importing country to unnecessary instability through erratic capital exports. An economy which needs a steady inflow of real capital so as to attain rapid growth over a given planning horizon would not welcome such short-run disturbances, although fluctuating capital imports may be better than no capital imports. It is not appropriate, either, in view of the fact that business cycles tend to spread internationally. When the U.S. economy booms so that its imports from underdeveloped areas flourish, the latter would have a brighter investment opportunity and attract foreign investors from advanced economies. When the U.S.A. becomes depressed, the latter must usually follow suit. In the final analysis, the counter-cyclical capital exports of the advanced economy tend to contribute to its stable growth at the relative cost of the capital-importing underdeveloped economy.

Let us proceed to consider other effects of capital exports on the advanced economy. Two important components of stable growth are price stability and full employment. The price-stability condition, identified as Harrod's 'steady-advance' condition, forestalls any inflationary or deflationary divergencies. The full-employment condition admits 4% unemployment so as to be consistent with the stability of the general

price level. The appropriate objectives are full employment and maximal consumption. All of the programmings (Chapter V) are subjected to two constraints, one on labor availability and the other on the ex-ante prevention of excess or short capacity. The former constraint can be replaced by the full-employment condition. The latter constraint incorporates capital exports. An increase in sectoral capital exports is reflected in a larger constant term of the steady-advance constraint. This fact demonstrates that the optimal value of the objective function can be improved through increased capital exports under certain favorable conditions. We now need to explain this observation in more detail. Exports of investment goods to underdeveloped countries are characterized by a fairly close connection between initial financial and subsequent real transfers. W. Guth states that "We could almost call exports of investment goods a function of medium and long term capital exports." The elasticity coefficient of Japan's heavy-industry goods exported to underdeveloped countries to Japan's financial assistance averages 0.3 . The industries of the advanced economy, especially those producing export-oriented investment goods (tractors, fertilizers, cranes, etc.) would receive more orders for their products than they would in the absence of the exports of the investment goods. This would set the process of multiplier-accelerator interaction in motion. So long as there are enough import demands in the recipient country for the goods and services of the donor country, a serious transfer problem would not take place. The resultant increase of income of the advanced economy brings an increase of labor and consumption demand. This explains the background of the above observation.

These optimizing models are based on the multisectoral growth model. Its advantage over the usual macro model lies in the fact that it enables the policy makers to select those sectors which are weak in their contributions to overall stable growth and adopt suitable selective policies. A word about the relationship between domestic and foreign investment may be added. If foreign investment occurs at the excessive cost of domestic investment, positive effects of the increased capital exports may not offset negative effects of the decreased domestic investment. This would typically be the case when the intersectoral mobility of resources is limited. So long as there exist unemployed productive factors or sufficient substitutability of resources between domestic and foreign investment sectors, the increase of capital exports, which is reasonable apropos of domestic investment demand, would (more than) offset possible disadvantageous effects of decreased domestic investment demand. In the context of Harrod-Domar-type growth theories which assume rigidities in resource mobility, the increase in capital exports must presuppose unemployed productive factors, so that it can absorb them in such a way as to attain net positive effects on employment and consumption. Under these circumstances the capital exports can contribute to attainment of full employment and hence to the stable growth of the advanced economy.

This observation based on the single-country model needs further qualifications when repercussive effects of the underdeveloped economy come into the picture (Chapter VI). In order for the advanced economy to increase its income (and hence its employment and consumption), a certain favorable combination of its endogenous coefficients is required.

It implies that either the capital coefficient or the growth rate of the trend output must be small enough relative to the saving ratio. Since a relatively low growth rate is desired in the advanced economy, so that its full employment position may be maintained with general price stability, the condition boils down to the fact that the relatively high capital-output ratio times the relatively low growth rate of the trend output must be smaller than the saving ratio. The marginal productivity of the capital exports then becomes positive, on the assumption that the favorable combination of the endogenous coefficients is attained on the part of the underdeveloped economy. This conclusion derived from the two-country model (Chapter VI) contrasts with the tentative conclusion derived from the simple single-country model (Chapter I), which is based on the simplifying assumption that overlooks indirect long-run and repercussive effects. The negative effect of the outflow of saving (capital) on the growth rate of the domestic equilibrium output can in itself mitigate inflationary pressures, if any, and thus contribute to stable growth, and yet it claims reconsideration in view of the overlooked aspects. It can be more than offset by the positive effects of capital exports on the level and the growth rate of the output. This point of view must be born in mind in considering the stabilizing role of capital exports. In reality, the net effect of capital exports on the growth rate of the advanced economy should be a compromise of the conclusion derived from the single-country model and that drawn from the more realistic two-country model.

A word about the comparative effect of the exports of goods and services on current account may be added before proceeding to our third

major problem. Harrod contends that the exports of goods and services on current account tend to decrease the growth rate of the domestic equilibrium output (Chapter I). For exports are considered as a leakage of production. His contention does indeed retain the same kind of usefulness as the outflow of saving (capital) in stabilizing the growth rate of the output, and yet is based on the simplifying cet. par. assumption to neglect all the repercussive and long-run effects of exports. The net effect of exports on the stable growth of the advanced economy must be judged by the extent of the dynamic export multiplier (the marginal ratio of the trend output to the exports) (Chapter VI) modified downward by the direct growth-rate reducing effect of the current-account exports (Chapter I). A high growth rate of current-account exports is in reality associated with a high rate of growth of GNP. The capital imports exert a positive influence on the growth rate of GNP. Trade and capital movement appear to be still complementary to each other. The tenet that factor and commodity movements are partial substitutes for each other is based on an over-simplified model, and fails to explain dynamic and long-run forces on which our analysis has focused. The argument relevant to economic development must be 'trade and aid,' rather than 'trade vs. aid.'

Our third major problem is the interacting effects of capital movements on the advanced and the underdeveloped economy. We derived the respective domestic conditions required for mutual gains through capital movements. Unless the two conditions specified in Chapter VI are simultaneously satisfied, both of the countries can not gain a benefit. Since they are of central importance to our whole study,

those necessary conditions are worth repeating: first, the investment ratio must exceed the saving ratio plus the import ratio on the part of the underdeveloped economy. Second, the saving ratio must exceed the investment ratio on the side of the advanced economy. In other words, either the capital-output ratio or the growth rate of the trend output must be sufficiently large, so that the former times the latter exceeds the saving ratio plus the import ratio in the underdeveloped economy. Either the capital-output ratio or the growth rate of the trend output must be sufficiently small, so that the former times the latter is exceeded by the saving ratio in the advanced economy. These are the results which we must reach, so long as we base our analysis on the initial key assumptions that the ex-ante domestic saving plus the ex-ante import must in equilibrium be equal to the ex-ante export plus the domestic and foreign (exported) ex-ante investment in the capital-exporting country, and that the ex-ante domestic investment plus the ex-ante export must in equilibrium be equal to the ex-ante import plus the domestic and foreign (imported) ex-ante saving in the capital-importing country. Notice that we did not assume the ex-post (or accounting) context that a country's net export surplus is identical to its capital export, which is also equal to its saving minus investment, and that a country's net import surplus is identical to its capital import, which is equal to is investment minus saving.

We are fairly confident of the plausibility of the two domestic conditions. The tendency of the saving ratio to exceed the investment ratio is a typical phenomenon of an advanced economy. This is the substance of the Hansen Stagnation Thesis. The advanced economy's

growth rate of output is in reality relatively low and stable. This would keep harmony with general price stability. In the case of the U.K., the postwar growth rate of output is 2.45%, while the aggregate capital-output ratio is 5.61, for the period 1951-1957. The relatively low potential growth rate combines with the capital-output ratio which is not very high, to result in an investment ratio short of the saving ratio in an ex-ante setting. Saving then is no longer a virtue, so long as the implied warranted growth rate exceeds the implied natural growth rate. Limited investment opportunities leave the investment funds idle and cause a depression in the domestic economy. Entrepreneurs look for overseas markets and make further foreign investments in some of the developing economies.

On the part of the underdeveloped economy, the domestic condition, the ex-ante excess of the investment ratio over the sum of the saving and the import ratio, symbolizes a typical symptom of the developing country. Its implied potentially inflationary tendency is mild enough and conducive to rapid development, so long as non-capital complementary factors and facilities are available. Saving is a virtue here, inasmuch as the implied natural growth rate exceeds the implied warranted growth rate. It underlies the Investment-Saving-Gap analysis of the capital needs of developing countries (as compared with their Import-Export-Gap analysis (Chapter VIII)). Since the growth rate of output and the capital-output ratio are inversely correlated, especially in the postwar world, the underdeveloped country's relatively high investment ratio offers two alternative interpretations: either a relatively low growth rate of output coupled with a relatively high capital-output

ratio, or a relatively high growth rate of output combined with a relatively low capital-output ratio. We sometimes observe the first alternative in underdeveloped countries. The actual relatively high capital-output ratios (with the relatively low growth rates) are explained by the structural need for the capital-intensive methods of production, the structural need for durable equipment (capital stocks are most deficient), or the low aggregate productivity of capital.

However, the relatively high capital-output ratio coupled with the relatively low growth rate of output is not the desideratum of a developing country. The policy makers aim at the second alternative: the relatively high growth rate of output combined with the relatively low capital-output ratio, as exemplified by the case of Jordan which attained the GNP growth rate 11.1% with the incremental capital-output ratio 1.36 for the period 1957-1962.

The target of the representative developing economy consists in a rapid growth. By contrast, the aim of the representative advanced economy lies in a stable growth. These ideas about the desired growth patterns underlie all the optimizing models of our study (Chapters III, V and VII). They are the derivatives of the preceding respective deterministic models. The recent tendency of economic theory in general lies in the movement from deterministic to optimizing models. We present two types of global programmings (Chapter VII) by combining conceptually the optimizing models for an underdeveloped economy (Chapter III) with those for an advanced economy (Chapter V). The first type is designed for finding the optimal combination of capital movements which would minimize the national-income gap between the two

unequal economies subject to the domestic growth requirements. The second type is intended to minimize the standard-of-living gap subject to the same constraints. These global optimizing models would enhance their practicability, provided that the two domestic conditions required for mutual gains through capital movements (Chapter VI) are guaranteed and that population growth is controlled to a desirable degree, especially in underdeveloped areas. These reservations need to be born in mind when we consider the feasibility of the conclusion derived from the global optimizing models: the gaps of the national incomes and the living standards can be narrowed by international capital movements (Chapter VII).

The relatively high growth rate of output ought to be tied to the relatively low capital-output ratio in the underdeveloped economy, on the assumption that the saving and the import ratio remain relatively unaffected. The aggregate productivity of capital must be enhanced. More labor-intensive methods of production would become necessary. Long-run conscious efforts are required in order to remove the road-blocks in the way, such as inadequate infrastructure, non-capital cooperative factors and facilities. Capital imports can not only enhance the level of the trend output but also help eliminate these obstacles, especially when accompanied by efficient technical assistance. They can also accelerate the growth rate of output (Chapter I). If, in addition, the domestic saving ratio is raised through domestic taxation or income redistribution policy, the attainment of the desired growth pattern becomes so much easier. Capital exports, in turn, can not only increase the level of the trend output but also help eliminate

the depressing situation emanating from excessive domestic saving and the deficient domestic inducement to invest. They can also lower but stabilize the growth rate of output (Chapter I).

In conclusion, international capital movements can enhance the outputs of both the advanced and the underdeveloped economy under the fairly plausible conditions stated above. They also tend to bring about the desired growth rate in the respective countries. Both the advanced and the underdeveloped country can attain the greatest mutual benefits through their domestic and international efforts to bring the favorable conditions into existence. The international movements of real capital, which are a sine qua non of these mutual gains and of a wider basis for realistic peace, must be carried out on an expanded scale.

Section 3

Limitations

A revaluation of a couple of the crucial assumptions, on which the whole study continues to be based, constitutes a useful postscript to the above conclusion. The aim of this postscript lies in clarifying the limitations of our study. It will also serve to suggest a direction of possible extension of this study. The adopted models, which are extensions of the Harrod-Domar-type growth theory, retain the assumption of fixed coefficients. Important among these are the fixed marginal propensity to save, the fixed marginal propensity to consume,

the fixed marginal propensity to import, and constant capital coefficients. The variability of these strategic coefficients, as conceived by neo-classical theorists, is left in shadow, owing to the observed rigidities of the economic structures of the advanced and the underdeveloped countries, and for the sake of argument. The present study is not quite relevant to an economy in which a continuous variability of these coefficients must be regarded as important. The assumed constant capital coefficients imply that the technical progress is here ignored or, at best, Harrod-neutral. The neo-classical assumption of factor substitutability based on perfect factor-price flexibility and factor non-specificity is thus bypassed. Although some of the programming models include labor as well as real capital, one-factor models dominate the picture. For real capital is regarded as the most important element in growth and development. Constant returns to scale are subsumed. No serious monetary analysis appears in the study; the price mechanism here does not play an important role, as it does in neo-classical models. This is part of the reason why we could separate the problem of a rapid growth from the problem of inflation. A detailed analysis of the relation between inflation and capital imports goes beyond the scope of this study. The function of price and monetary factors is particularly doubted in an economy with underdeveloped market systems.

Along with these assumptions essential to Harrod-Domar-type models, the following assumptions are made in connection with capital movements. Interest rates, dividends and amortization payments are negligible. The function of interest rates is especially negligible in the case of

capital movements between advanced and underdeveloped economies. The interest rates are not necessarily higher in capital-poor underdeveloped countries than in capital-rich advanced countries. Moreover, capital does not always move from countries with low interest rates to those with high interest rates (Arndt). A counter argument might point out the importance of interest, dividends, and amortizations from the viewpoint of the balance of payments. The answer would be that the balance of payments problem often tends to be stressed too far from a rather short-run standpoint. It is less important than the problem of rapid growth. As long as the developing economy is capable of achieving a higher rate of growth more by capital imports than by scanty domestic saving, there is no reason why it should not initially receive capital from abroad. In other words, emphasis should be on domestic equilibrium growth rather than on external payments equilibrium (Chapter I). This is the process which the 19-th century U.S. economy or the postwar Japanese economy followed. Later, when the repayment period arrives, it will have little trouble under the favorable conditions specified above. In the context in which these favorable conditions are difficult to be satisfied, our analysis retains its validity only for the aid-type capital, i.e., pure gifts or, at least, capital with very generous repayment conditions (as preferably without any political strings).

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