Prior Analytics and Aristotle's Commitment to Logos

George Boger
Canisius College, boger@canisius.edu

Follow this and additional works at: https://orb.binghamton.edu/sagp

Part of the Ancient History, Greek and Roman through Late Antiquity Commons, Ancient Philosophy Commons, and the History of Philosophy Commons

Recommended Citation

This Article is brought to you for free and open access by The Open Repository @ Binghamton (The ORB). It has been accepted for inclusion in The Society for Ancient Greek Philosophy Newsletter by an authorized administrator of The Open Repository @ Binghamton (The ORB). For more information, please contact ORB@binghamton.edu.
Prior Analytics and Aristotle’s commitment to logos
George Boger, Canisius College. 28 December 1996. APA Eastern Meeting, Atlanta, GA

1. Aristotle’s commitment to logos. It is evident from the character of his extensive philosophical writings that Aristotle was animated by a profound commitment to logos and to the centrality of reason in human life. We can, first of all, recognize Aristotle’s commanding commitments to discovering truth and to establishing knowledge, whether in the natural sciences, in metaphysics, or in ethics and politics. While “all men by nature desire to know”, Aristotle is exemplary in aspiring to realize wisdom, to possess genuine “knowledge of causes and first principles”. It is little wonder that St. Thomas was fond of calling him “the philosopher”. Secondly, we see that Aristotle expressed his commitment to logos by affirming the principle that nature in its diversity and human beings in their complexity are indeed comprehensible. He writes in Metaphysics 1.2:

The acquisition of this knowledge [epistêmê] may rightly be regarded as not suited for man. ... God may alone have this prerogative, and it is fitting that a man should seek only such knowledge as becomes him [and not, as the poets say, arouse the gods’ jealousy]. But we should not believe in divine jealousy; for it is proverbial that bards tell many lies, and we ought to regard nothing more worthy of honor than such knowledge. ... This science alone may be divine, and in a double sense: for a science which God would most appropriately have is divine among the sciences; and one whose object is divine, if such there be, is likewise divine. (982b28-983a7; emphasis added)

Perhaps Aristotle can be appreciated in this respect as sharing the Enlightenment’s spirit of daring to know and its optimistic confidence in reason’s ability to establish objective knowledge. We can see, furthermore, from the mission statement of the Analytics “to inquire about demonstration and about demonstrative science” (24a10-11), that Aristotle was committed to re-presenting the truth intelligibly and honestly. Posterior Analytics in particular is concerned with the organization and presentation of the results of scientific study — each science is to be axiomatized in order to make its subject matter intelligible and thus accessible. Aristotle remarks in Metaphysics 1.2 that ‘wisdom pertains to scientific knowledge of that which is most intelligible, namely, of causes and first principles, since it is through them that any given subject matter becomes intelligible’. From this we can understand his other remarks there that “a sign of one who knows is his ability to teach” and that “a theoretical science of causes is the most educative”. Again, in Nicomachean Ethics 6.3 he writes that “every science seems to be teachable, and what is scientifically knowable is learnable” (1139b25-26). Apprehending the truth, then, the philosopher in his commitment to logos also is educator.

In spite of his complaints about Socrates, it is nevertheless evident that Aristotle embraced Socrates’ teaching in the Phaedo (89d) that “there is no worse sin than mistología” and in the Apology (38a) that “the unexamined life is not worth living for mankind”. Aristotle, perhaps more than others, grasped the importance of reason. The lessons in Nicomachean Ethics commit us to a life of reason as essential for achieving happiness and for realizing our very humanity. Human virtue consists in making excellent the soul’s deliberative and scientific faculties. Phronêsis “is a state grasping the truth, involving reason [logos], concerned with action about what is good and bad for a human being” (NE 6.5: 1140b4-6). Sophia “is understanding (nous) plus scientific knowledge (epistêmê) of the most honorable things” (NE 6.7: 1141a18-20). Happiness, the life of philosophy, is a realization of the divine element in human nature. When we consider Aristotle’s bold statements in De Anima 3.5 and 3.7 that “actual knowledge is identical with its object” (430a19-20; 431a1-2), we can more fully appreciate his exhortation in NE 10.7 that reaffirms the spirit of his inquiry in Metaphysics 1.2: “We ought not to follow the proverb-writers to ‘think moral thoughts’ ... Rather, as far as we can, we ought to strive to be immortal and to go to all lengths to live a life that expresses our supreme element” (1177b31-34). Prometheus stole for us fire from the hearth of the Olympians; Aristotle secured for us a place at their table.

2. In John 1.1 we read that “in the beginning was the logos”. Herakleitos reports that “the logos is common” and that “all things happen according to the logos”. With Parmenides we might say that in the beginning “whatever is, is”. The first philosophers recognized that the truth is out there and that it is discoverable. Only later is there knowledge of the truth as the thinking subject puzzles about the object world and comes to know the truths that ‘nature loves to hide’. On the way to knowledge, however, there are many obstacles, dead ends, and false persuasions against which we must be instructed — the path may seem ‘backward turning’ and ‘helplessness may guide the wandering thoughts of mortals’. Too many misguided thinkers ‘confuse an apparent harmony for a better, unapparent harmony and
misapprehend the truth’. While the preplatonic philosophers were concerned with saving appearances by discovering underlying realities, only the Academicians, perhaps, undertook real efforts to develop a systematic method. Human beings, they early realized, need an instrument to help them separate what is true from what is false and erroneous, what is knowledge from what is mere opinion. Humans require a method to help eliminate error and to establish truth. Aristotle took up the torch and carried the project through to its first completion. In Sophistical Refutations 34 (183b38-184a3) he remarks on his own grand accomplishment in this respect: “earlier teachers of rhetoric and dialectic trained their students by having them learn discourses by heart. Hence their teaching was rapid but unsystematic. ... However, we had no earlier work to refer to concerning a systematic account of syllogistic reasoning”.

Now, the philosopher tells us in Posterior Analytics A1 that all knowledge comes from previous knowledge and generally in two ways: through epagôgê or induction and through sullogismos or deduction. This suggests two aspects of human knowledge. On the one hand, there is the what that is known concerning a given subject matter. The what has an ontic nature and pertains to the object world. On the other hand, there is the how things are known; this pertains to the means or instrument for processing information about a given subject matter. The how has an epistemic nature. The first is a topic specific and contentful matter having to do with substances relating to a given universe of discourse. The second is a topic neutral and formal matter having to do with method, that is, with the concerns of logic and deduction systems. We may now add an important corollary to Aristotle’s commitments to logos: to establish a deduction system that serves to eliminate error and to establish scientific knowledge. Prior Analytics fulfills this commitment.

However contemporary scholars may variously account for the ancient accomplishments, we can nevertheless observe their keen interest to systematize knowledge. Notable in this respect are Euclid’s Elements for geometry and Aristotle’s outlining in Prior and Posterior Analytics the requirements for demonstrative science (apodeiktikê epistêmê). We may take these latter works as especially representative of Aristotle’s commitments to logos and to humanity. Part of his genius is to have been the father of formal logic, of having created the science of logic which is decidedly a metalogical activity. The difference between logic and metalogic may be drawn as that between using an underlying logic to process information about a given subject matter in a given object language, on the one hand, and studying, on the other hand, an underlying logic, which consists in a language, a semantics, and a deduction system. Logics themselves are the objects of metalogic or the science of logic just as animals are the objects of zoology. In this connection, then, we take Prior Analytics to be a metalogical study of the syllogistic deduction system, which, taken with Categories and De Interpretatione, comprises Aristotle’s treatment of an underlying logic.

3. A new interpretation of Prior Analytics. Our interpretive standpoint takes Aristotle’s purpose in writing Prior Analytics as providing a proof-theoretic discourse that verifies the results of his consciously (1) constructing a natural deduction system for the purpose of deducing theorems from principles and (2) demonstrating certain of that system’s properties. Aristotle realized a remarkable metalogical sophistication in his service to truth and in his commitment to logos.

Three interpretive trends generally characterize studies of Aristotle’s logical investigations. The traditionalist interpretation holds that Prior Analytics is a handbook for studying categorical arguments or syllogisms. Proponents of this view such as J. N. Keynes, R. M. Eaton, and W. D. Ross usually treat a sullogismos as a fully interpreted valid or invalid premise-conclusion argument1 consisting in three categorical sentences -- two and only two premises and one conclusion -- whose validity or invalidity is ascertained by the familiar rules of the syllogism, namely, those of quality, quantity, and distribution. The following are typical examples of (first figure) syllogisms from the traditionalist standpoint.

A VALID SYLLOGISM

1. Every reptile is an animal.
2. Every turtle is a reptile.
? Every turtle is an animal.

AN INVALID SYLLOGISM

1. Every man is an animal.
2. No horse is a man.
? Every horse is an animal.

1 An argument is a two part system, consisting in a set of sentences in the role of premises and a single sentence in the role of conclusion, and is either valid or invalid. The ‘?’ used in this paper indicates the conclusion of an argument. Cf. J. Corcoran 1989 on definitions of logic terminology.
The axiomaticist interpretation, originating in the 1920s and introducing the use of mathematical logic to study Aristotle, takes *Prior Analytics* to lay out an axiomatized deductive system of theorems analogous to Euclid's *Elements*. J. Lukasiewicz, I. M. Bochenski, and G. Patzig take a *sullogismos* to be a single, relatively uninterpreted, logically true conditional proposition — the antecedent conjoins two categorical propositions, the consequent is a single categorical proposition. A *sullogismos* on this account is a logical law and not an argument. Some *sullogismoi* are axioms and others are theorems derived from those axioms according to a propositional logic used only implicitly by Aristotle. These interpreters admit that Aristotle did not himself axiomatize his syllogistic system. The following are examples of syllogisms according to axiomaticists.

<table>
<thead>
<tr>
<th>FIRST FIGURE</th>
<th>SECOND FIGURE</th>
<th>THIRD FIGURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>[(AaB) &amp; (BaC) \Rightarrow (AaC)]</td>
<td>[(MaN) &amp; (MeX) \Rightarrow (NeX)]</td>
<td>[(PaS) &amp; (RaS) \Rightarrow (PiR)]</td>
</tr>
<tr>
<td>[(AeB) &amp; (BaC) \Rightarrow (AeC)]</td>
<td>[(MaN) &amp; (MoX) \Rightarrow (NoX)]</td>
<td>[(PoS) &amp; (RaS) \Rightarrow (PoR)]</td>
</tr>
</tbody>
</table>

The deductionist interpretation, perhaps prevailing since the early 1970s, also using the theoretical apparatus of mathematical logic, argues that in *Prior Analytics* Aristotle was concerned with epistemics and a natural deduction process. J. Corcoran, T. Smiley, and R. Smith consider a *sullogismos* to be a deduction, that is, a fully interpreted argumentation having a cogent chain of reasoning in addition to premises and conclusion. For deductionists, a *sullogismos* may consist in a premise-set of two or more categorical propositions. They model Aristotle’s logic as a natural deduction system to make their case, but they think that Aristotle could not himself have modeled his system because, they hold, he did not distinguish logical syntax and semantics. The following is an example of a *sullogismos*, in particular, a perfect or perfected *sullogismos* (J. Corcoran 1974: 109), according to deductionists.

1. No reptile is a being having hair.
2. Every mammal is a being having hair.
3. Some mammal is an aquatic being.
? Some aquatic being is not a reptile.

4. No being having hair is a reptile.
5. Every mammal is a being having hair.
6. No mammal is a reptile.
7. Some aquatic being is a mammal.
8. Some aquatic being is not a reptile.

1 e-conversion
2 repetition
4,5 Célerent
3 i-conversion
6,7 Ferio

Still, notwithstanding these important differences, the three interpretations share remarkably similar views on some matters central to Aristotle’s logic, notably (1) that concerning the nature of reduction in his system and (2) that concerning his methods for establishing invalidity. Concerning reduction, the different interpretations mistakenly conflate three processes that Aristotle explicitly distinguishes: (1) the completion or perfection (*teleiosis* or *to teleiousthai*) of *sullogismoi* at *A4-6*; (2) the reduction (*anagôgê* or *to anagein*) of *sullogismoi* at *A7* (*A23*); and (3) the analysis or resolution (*analusis* or *to analuein*) of *sullogismoi* at *A45*. On the matter of invalidation, interpreters mistakenly take Aristotle (1) to consider arguments or deductions and (2) to use the method of counterargument. However, Aristotle neither treats arguments or deductions *per se* in *Prior Analytics* *A4-6* nor anywhere explicitly uses the method of counterargument. Rather, he principally uses what we designate the method of counter contrariety to invalidate premise-pair patterns and their corresponding argument patterns.  

---

2 J. Lukasiewicz takes Aristotle's letters 'A', 'B', 'C', 'M', 'N', 'X', 'P', 'R', and 'S' to be non-logical variables for which only universal terms may be substituted (see J. Lukasiewicz 1958: 7-9; cf. G. Patzig 1968: 12-13). The 'a', 'e', 'i', and 'o' are our abbreviations for Aristotle's logical constants (see below section 4 and n7). The '&&' and '=>' are symbols (not Lukasiewicz's notation) for familiar logical constants of propositional logic.

3 An *argumentation* is a three part system consisting in a chain of reasoning in addition to premises and conclusion and is either cogent, in which case it is a *deduction*, or fallacious, in which case it is a *fallacy*.

4 An *argument pattern* is a two part system consisting in a set of sentence patterns in a role analogous to the premise-set of an argument and a single sentence pattern in the analogous role of conclusion. An argument is said to fit or to be an
We hold that Aristotle constructed a deduction system and himself modeled this logic in order to perfect the system and to determine certain of its mathematical properties. Accordingly, then, Aristotle is seen at Prior Analytics A4-7 as exhaustively treating only and all possible combinations of premise-pair patterns to demonstrate which pair patterns are concludent or generate sullogismoi and which pair patterns are inconcludent or do not generate sullogismoi. In this connection, we hold, contrary to previous beliefs, that a sullogismos as treated in A4-7 is neither a valid or invalid premise-conclusion argument, nor a single, logically conditional proposition, nor a deduction. Rather, as a relatively uninterpreted object, it is an elemental argument pattern, in one of three figures, consisting in three sentence patterns with places for three terms -- there are a premise-set of only two categorical sentence patterns and a conclusion of a single categorical sentence pattern. The following are schematic representations of the first figure or teleioi sullogismoi on our interpretation.5

<table>
<thead>
<tr>
<th>BARBARA</th>
<th>CELARENT</th>
<th>DARII</th>
<th>FERIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>∴ AaC</td>
<td>∴ AeC</td>
<td>∴ AiC</td>
<td>∴ AoC</td>
</tr>
</tbody>
</table>

Moreover, a sullogismos is an argument pattern all of whose instances are valid arguments. Aristotle recognized the epistemic efficacy of such elemental panvalid argument patterns and formulated them as rules of deduction in corresponding sentences.6 His treatment of syllogistic logic is precisely analogous to the activities of modern logicians and philosophers of logic who, in their studies of propositional logic, distinguish between the subject matter of a discourse and the formal or syntactic character of the deduction system used to process information relating to the subject matter. Thus, for example, just as ‘modus ponens’ and ‘disjunctive syllogism’ name elemental panvalid argument patterns which serve as familiar rules of deduction for propositional logic and are treated as syntactic objects, so ‘Barbara’ and Celarent’ name rules for Aristotle’s syllogistic logic and are treated by him in a similar fashion.

4. Aristotle’s natural deduction system. In Prior Analytics A4-7 Aristotle establishes a set of deduction rules as part of his natural deduction system. This system consists in four kinds of categorical sentence, two pairs of contradictories and one pair of contraries, three conversion rules, four sullogismos rules, and direct and indirect proofs. We outline the system as follows:

There are four LOGICAL CONSTANTS for the four kinds of categorical sentence:

| to panti huparchein | belongs to every | a | P belongs to every S | PaS |
| to mëdeni huparchein | belongs to none  | e | P belongs to no S    | PeS |
| to tini huparchein  | belongs to some  | i | P belongs to some S  | PiS |
| to më tini huparchein| belongs not to some | o | P belongs not to some S | PoS |

instance of a given argument pattern. An argument pattern is not properly valid or invalid. Rather, an argument pattern with all valid instances is panvalid, that with all invalid instances is paninvalid, and that having instances of both is neutrovalid. A pattern is commonly represented schematically. See. J. Corcoran 1993.

5 Here the ‘A’, ‘B’, ‘C’ are Aristotle’s schematic letters; the ‘a’, ‘e’ ‘i’ and ‘o’ are our abbreviations of Aristotle’s logical constants. The ‘::’ indicates a categorical sentence pattern that has been established as following from a given premise-pair pattern. See below section 4 on the use of characters here and elsewhere in this paper.

6 Consider, for example, Aristotle’s treatment of Barbara and Celarent -- the rule is stated at 25b32-35; the argument patterns are represented schematically at 25b37-39 and 25b40-26a2 -- and of Darii and Ferio -- the rule is stated at 26a17-20; the argument patterns are represented schematically at 26a23-25 and 26a25-27. Aristotle treats the second and third figure sullogismoi in the same manner.
* There is a standard syllogistic syntax:\(^7\)

<table>
<thead>
<tr>
<th>First figure: PMS</th>
<th>Second figure: MPS</th>
<th>Third figure: PSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(PxM, MxS \mid PxS)</td>
<td>(MxP, MxS \mid PxS)</td>
<td>(PxM, SxM \mid PxS)</td>
</tr>
<tr>
<td>major premise</td>
<td>1. (PxM)</td>
<td>1. (MxP)</td>
</tr>
<tr>
<td>minor premise</td>
<td>2. (MxS)</td>
<td>2. (MxS)</td>
</tr>
<tr>
<td>conclusion</td>
<td>(\therefore ) (PxS)</td>
<td>(\therefore ) (PxS)</td>
</tr>
</tbody>
</table>

* There are three pairs of opposite sentences:

**Two pairs of contradictories**

<table>
<thead>
<tr>
<th>(PaS) &amp; (PoS)</th>
<th>(PeS) &amp; (PiS)</th>
</tr>
</thead>
</table>

**One pair of contraries**

| \(PaS\) & \(PeS\) |

* There are three conversion rules, two simple conversions, one per accidens conversion:

\[\begin{align*}
& e\text{-conversion} & i\text{-conversion} & a\text{-conversion per accidens} \\
1. \(PeS\) & 1. \(PiS\) & 1. \(PaS\) & \(\vdash SiP\) \\
\vdash \(SeP\) & \vdash \(SiP\) & \\
\]

* There are four teleioi sullogismoi:\(^8\)

<table>
<thead>
<tr>
<th>Barbara</th>
<th>Celarent</th>
<th>Darii</th>
<th>Ferio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (AaB)</td>
<td>1. (AeB)</td>
<td>1. (AaB)</td>
<td>1. (AeB)</td>
</tr>
<tr>
<td>2. (BaC)</td>
<td>2. (BiC)</td>
<td>(\vdash (AiC)</td>
<td>(\vdash (AoC)</td>
</tr>
</tbody>
</table>
| \(\vdash \(AaC\) | \(\vdash \(AeC\) | \\

* There are two kinds of deduction:

**Direct deduction (Cames: \(A5, 27a9-14\))**

<table>
<thead>
<tr>
<th>1. (MaN)</th>
<th>2. (MeX)</th>
<th>? (NeX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. (MeX) &amp; 2 repetition</td>
<td>4. (XeM) &amp; 3 (e\text{-conversion})</td>
<td>5. (MaN) &amp; 1 repetition</td>
</tr>
<tr>
<td>6. (XeN) &amp; 4, 5 (Celarent)</td>
<td>7. (NeX) &amp; 6 (e\text{-conversion})</td>
<td></td>
</tr>
<tr>
<td>(3. (NaX) &amp; assume</td>
<td>4. (MaN) &amp; 1 repetition</td>
<td>5. (NaX) &amp; 3 repetition</td>
</tr>
<tr>
<td>6. (MaX) &amp; 4, 5 (Barbara)</td>
<td>7. (MoX &amp; MaX) &amp; 2, 6 (conjunction X)</td>
<td>8. (NoX) &amp; 3-7 (reductio)</td>
</tr>
</tbody>
</table>

**Indirect deduction (Bar: \(A5, 27a36-27b1\))**

<table>
<thead>
<tr>
<th>1. (MaN)</th>
<th>2. (MeX)</th>
<th>? (NeX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. (MeX) &amp; 2 repetition</td>
<td>4. (XeM) &amp; 3 (e\text{-conversion})</td>
<td>5. (MaN) &amp; 1 repetition</td>
</tr>
<tr>
<td>6. (XeN) &amp; 4, 5 (Celarent)</td>
<td>7. (NeX) &amp; 6 (e\text{-conversion})</td>
<td></td>
</tr>
<tr>
<td>(3. (NaX) &amp; assume</td>
<td>4. (MaN) &amp; 1 repetition</td>
<td>5. (NaX) &amp; 3 repetition</td>
</tr>
<tr>
<td>6. (MaX) &amp; 4, 5 (Barbara)</td>
<td>7. (MoX &amp; MaX) &amp; 2, 6 (conjunction X)</td>
<td>8. (NoX) &amp; 3-7 (reductio)</td>
</tr>
</tbody>
</table>

Our discussion below works with this understanding of Aristotle's treatment of an underlying logic.

---

\(^7\) Aristotle in *Prior Analytics* uses strictly a standard syllogistic syntax when he considers argument patterns relating to given premise-pair patterns. Each such pattern consists in three different terms in combinations of the four kinds of categorical sentence. He always understands the predicate term \([P]\) of the conclusion to be the major term in the premise-pair pattern, the subject term \([S]\) of the conclusion to be the corresponding minor term. The term common in the premises is the middle term \([M]\) whose relationship to the other two terms is determinable in three figures, first, second, and third: respectively, \(PMS, MPS, PSM\). Using \('P', 'S', and 'M'\) as schematic letters (neither term variables nor abbreviations for term constants), we can express the pattern of every categorical sentence schematically as \(PxS\) (where \('x'\) ranges over Aristotle's four logical constants). In the standard syllogistic syntax the conclusion always fits the sentence pattern \(PxS\) and never its converse. In discussing Aristotle's treatment of premise-pair and argument patterns we use the following schematic notation: in the expression, for example, \('PaM, MaS \mid PaS'\) the \('PaM, MaS \mid '\) signifies the premise-pair pattern of two categorical sentences separated by \(', '\) (a comma) and distinguished as premises by \('|' (a vertical bar), which character is followed by a sentence pattern in the place of a conclusion.

\(^8\) These four patterns are reduced to the two universal teleioi sullogismoi at \(A7\). See below section 5.3.
5.0. **Aristotle's metalogical study of syllogistic reasoning.** Aristotle had genuine **proof-theoretic interests** to establish theorems about his system. At A4-6 Aristotle determines “how every sullogismos is generated” (25b26-31). He exhaustively treats only and all possible combinations of premise-pair patterns. He thereby determines which pair patterns generate a sullogismos and are panvalid with no counter instances and, by a method using a principle of syllogistic contrariety, which pair patterns do not generate a sullogismos and are paninvalid. (No pattern is neutroválid.) A sullogismos is treated exclusively as having only three terms and two premises; this is emphatically stated at A25 (see esp. 41b36-37, 42a30-31, 32-33). Once having identified all the sullogismoi he proceeds in A7 to eliminate redundancy and in A45 to establish their inter-relationships. Below we identify four metalogical processes relating to Aristotle's rules of deduction: completion, invalidation, reduction, and analysis.

5.1. **Completion at A4-7.** Each of the four first figure sullogismoi is teleios, perfect or complete: that is, the necessity of the result following from the things initially taken is **immediately evident**, nothing additional need be taken (A1, 24b18-24; A4, 26b28-33). Aristotle does not prove in A4 that certain premise-pair patterns result in teleioi sullogismoi -- they are just posited or understood as having a logical consequence “di’ hautôn” or through themselves. Each of the four second figure and the six third figure sullogismoi is atelès, imperfect or incomplete. Their necessity is not immediately evident but dunatos, that is, potentially evident. Thus, a **deduction is required** to make the necessity evident (see: A1, 24b24-26; A5, 27a1-3, 27a15-18, 28a4-7; A6, 28a15-17, 29a14-16). In each case of a second and a third figure sullogismos Aristotle shows by means of a metalogical deduction that a given premise-pair pattern generates a sullogismos (viz. an atelès sullogismos). At A5-6 Aristotle uses his natural deduction system to **demonstrate** that a given second or third figure argument pattern is, in fact, a sullogismos. The text concerning Camestres illustrates this.

If M belongs to every N but to no X, then neither will N belong to any X. For if M belongs to no X, neither does X belong to any M; but M belonged to every N; therefore, X will belong to no N (for the first figure has again been generated). And since the privative converts, neither will N belong to any X. (27a9-14)

We can express exactly what Aristotle writes here in the manner of a deduction familiar to us.

| 1. | MaN  |
| 2. | MeX  |
| ? | NeX  |
| 3. | MeX  | 2 | repetition |
| 4. | XeM  | 3 | e-conversion |
| 5. | MaN  | 1 | repetition |
| 6. | XeN  | 4, 5 | Celarent |
| 7. | NeX  | 6 | e-conversion |

Aristotle obviously uses the first figure sullogismoi as rules of deduction in A5-6, and he explicitly mentions their use there in the proofs of Cesare (27a12-13), Festino (27a36), Darapti (28a22), and Ferison (28b34-35). Aristotle’s summary of A4-6 at A7 highlights this point:

all the incomplete sullogismoi are completed by means of [teleiounthai dia] the first figure. For they all come to a conclusion [perainontal] either probatively or through an impossibility, and in both ways the first figure is generated. (29a30-36; cf. A5, 28a4-7 and A6, 29a14-16)

At A4-6 Aristotle establishes the preeminence of the first figure by positing the teleioi sullogismoi as rules of deduction at A4 and by using them as such at A5-6. Thus, implicitly he eliminates the atelès sullogismos as redundant rules of deduction for his deduction system. The verbs ‘teleiousthai’ and

---

9 Aristotle’s syllogistic functions at two levels of discourse, at the levels of logic and of metalogic. This has caused considerable interpretive confusion (see, e.g., G. Patzig 1968: 145). In Prior Analytics he not only studies a deduction system but he also uses that system metalogically to establish parts of the system itself.
‘epiteleisthai’ (to be completed) are synonyms as used at A4-7; they denote a process using the *teleioi sullogismoi* as rules.10

**Completion** is a proof-theoretic process that establishes knowledge that a sentence fitting a given sentence pattern results necessarily from two other sentences fitting given sentence patterns by means of using the *teleioi sullogismoi* as rules of deduction. Completion here is a metalogical deduction process whose epistemic import is to establish knowledge of panvalidity. This process does not eliminate redundant rules of deduction; nor does it involve transforming a given argument pattern into another argument pattern. Aristotle’s metalogic theorem concerning completion is that “all the incomplete *sullogismoi* are completed by means of the first figure *sullogismoi* using probative and *reductio* proofs” (A7, 29a30-33).

5.2. **Invalidation at A4-6.** Aristotle uses the method of counter contrariety at A4-6 to demonstrate which premise-pair patterns do not result in a *sullogismos* and are paninvalid. He writes:

However, if the first extreme follows all the middle and the middle belongs to none of the last, there will not be a *sullogismos* of the extremes, for **nothing necessarily results** in virtue of these things being so [ouden gar anankaion sumbainei tō(i) tauta einai]. (26a2-5; emphasis added)

This passage states a rule (a counterpart of those for the *sullogismoi*; see above n6) concerning the premise-pair pattern ‘*PaM, MeS*’; that **no sentence is a logical consequence of two sentences in the given pattern.** He continues:

For **it is possible** [endechetai] for the first extreme to belong to all as well as to none of the last. Consequently, neither a particular nor a universal conclusion results necessarily; and, since nothing is necessary because of these, there will not be a *sullogismos*. Terms for belonging to every are animal, man, horse; for belonging to none, animal, man, stone. (26a5-9; emphasis added)

Aristotle uses neither the method of counterargument nor the method of counterinterpretation, each of which requires finding an instance of an argument having true premises and a false conclusion in the same form11 as a given argument. Rather, by substituting terms for the schematic letters *A, B, C* (implicit at 26a2-9), he constructs two arguments each of whose premises are true sentences and fitting the same premise-pair pattern and whose conclusions also are true sentences, but in the one argument it is an *a* sentence, in the other an *e* sentence. Aristotle cites two sets of three terms which he substitutes into two argument patterns each with the same given premise-pair pattern. We can express what he says here, with truth values given in parentheses, as follows:12

<table>
<thead>
<tr>
<th>PaM, MeS</th>
<th>PaS</th>
<th>PaM, MeS</th>
<th>PeS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animal  a man. (T) AaM</td>
<td>1. Animal  a man. (T) AaM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Man  e horse. (T) MeH</td>
<td>2. Man  e stone. (T) MeS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We call such sentences, as given in the conclusions here, **imperfect syllogistic contraries**, in contrast to **perfect syllogistic contraries** that pertain to concludent premise-pair patterns, since it is logically

---

10 While ‘*teleiousthai*’ and ‘*epiteleisthai*’ are synonyms, ‘*perainesthai*’ (to bring to a conclusion), ‘*deiknusthai*’ (to be proved), and ‘*sullogizesthai*’ (to be syllogized) are not synonymous with them, nor even with each other.

11 We distinguish *form* from *pattern* as follows. While every sentence or argument has only one form, a sentence or an argument may fit more than one pattern. Thus, for example, the diversity of arguments, each having just one form, may all be said to fit the pattern “premise-set, conclusion” or, schematically ‘P-c’, where the ‘P’ holds the place for a premise-set and the ‘c’ the place for a conclusion.

12 The argument patterns are given at the top using ‘*P*, ‘*M*, ‘*S*’ as schematic letters for non-logical terms. The ‘*A*, ‘*M*, ‘*H*, and ‘*S*’ in the two given arguments are abbreviations for the non-logical constants. This way of representing the matter applies also to the table given in n13.
possible for both sentences to be true.\textsuperscript{13} For Aristotle this demonstrates that “nothing results necessarily” from sentences is this premise-pair pattern since, as he shows, the results “could be otherwise”. Thus, any sentences of three terms fitting this premise-pair pattern are shown never to result together in a valid argument -- there is no \textit{sullogismos} of the extremes through the middle. This premise-pair pattern is inconclusive. It is evident, moreover, that he treats \textit{at one time} in this way the four possible argument patterns in the standard syllogistic syntax for each premise-pair pattern. With 26a5-9 Aristotle establishes a practice that he uses throughout \textit{A4-6} to demonstrate inconclusiveness: he selects terms to provide imperfect syllogistic contraries to show that nothing results necessarily from a given premise-pair pattern. The expression ‘counter contrariety’ denotes this situation.\textsuperscript{14}

THE METHOD OF COUNTER CONTRARIETY is a proof-theoretic process by which a given premise-pair pattern is determined to be inconclusive and its corresponding argument patterns are determined to be paninvalid. Accordingly, he demonstrates not the invalidity of arbitrary arguments but (1) the inconclusiveness of premise-pair patterns and, consequently, (2) the paninvalidity of the four argument patterns associated with each inconclusive pair in the standard syllogistic syntax. This method eliminates certain argument patterns as possible rules of deduction.

5.3. Reduction at \textit{A7}. Commentators have conflated\textsuperscript{15} Aristotle’s treatment of the reduction of \textit{sullogismoi} at \textit{A7} with his treatment of their completion at \textit{A5-6} since in both cases he refers to the process of completion. However, in \textit{A5-6} he shows which argument patterns are \textit{sullogismoi} by proving that a conclusion follows necessarily by performing deductions using all four \textit{teleioi sullogismoi}, while at \textit{A7} he shows that the same argument patterns can be shown to be \textit{sullogismoi} using only the two universal \textit{teleioi sullogismoi}. At \textit{A7} Aristotle is decidedly concerned to demonstrate which patterns are \textit{sullogismoi} by using only the two universal \textit{teleioi sullogismoi} as rules of deduction. Reduction is portrayed here as a process from an \textit{ateles sullogismoi} to one of the two universal \textit{teleioi sullogismoi}. No \textit{sullogismoi} is reduced to a second or to a third figure \textit{sullogismoi}. By enumerating each possible case Aristotle shows that only Barbara and Celarent are \textit{sullogismoi} necessary for his deduction system.

\textbf{REDUCTION} is a proof-theoretic process that eliminates redundancy and establishes the independence of rules of deduction by showing that the same deductive results can be obtained using only the two universal \textit{teleioi sullogismoi}. At \textit{A7} Aristotle reaffirms that the second and third figure \textit{sullogismoi} are redundant as rules of deduction. By showing that the particular \textit{teleioi sullogismoi} (viz. Darii and Ferio) are not necessary, he also establishes the independence of the two universal \textit{teleioi sullogismoi} (viz. Barbara and Celarent) as among the rules of his deduction system. Reduction is not a method of validating arguments or argument patterns, which was already accomplished at \textit{A5-6}: it is not a substitute for syllogistic deduction. Nor is it a process for deriving some \textit{sullogismoi} from others to form a deductive system: it is not a process for axiomatizing a system of logic. Nor is reduction concerned with transforming arguments or argument patterns into one another. \textit{A7} treats the relationships among \textit{sullogismoi} taken as rules. Aristotle’s theorem concerning reduction is that “all the \textit{sullogismoi} can be reduced to the two universal \textit{sullogismoi} in the first figure” (\textit{A7}, 29b1-2).

\textsuperscript{13} \textit{Syllogistic contraries} consist in (1) two universal sentences -- the one an \textit{a} sentence, the other an \textit{e} sentence, (2) each sentence having a different predicate or subject term respectively, and (3) each sentence relating the extremes of two sets of three terms in two true sentences fitting a given premise-pair pattern. Such contraries are \textit{perfect} when it is logically impossible for both sentences to be true. The following example, relating to Barbara, illustrates this condition.

\begin{align*}
\text{PaM, MaS} & \quad \text{PaS} \\
1. \text{Animal} & \quad a \quad \text{mammal.} & \quad \text{T} & \quad \text{AaM} & \quad 1. \text{Plant} & \quad a \quad \text{tree.} & \quad \text{T} & \quad \text{PaT} \\
2. \text{Mammal} & \quad a \quad \text{horse.} & \quad \text{T} & \quad \text{MeH} & \quad 2. \text{Tree} & \quad a \quad \text{maple.} & \quad \text{T} & \quad \text{TeM} \\
? \text{Animal} & \quad a \quad \text{horse.} & \quad \text{T} & \quad \text{AaH} & \quad ? \text{Plant} & \quad e \quad \text{maple.} & \quad \text{F} & \quad \text{PeM}
\end{align*}

\textsuperscript{14} The method of counter contrariety works for almost all premise-pair patterns, noticeably failing in some instances when the minor premise is a particular sentence. In these cases Aristotle uses a modified method of counter contrariety (see e.g. 26b3-10) or a method of deducing inconclusiveness from the indeterminate (see e.g. 26b14-20).

5.4. Analysis at A45. Aristotle does not treat analysis at A45 in the same way as reduction at A7 or as completion at A5-6 in terms of completing (teleiousthai) or bringing to a conclusion (perainesthai). He has no concern with a deduction process here. Thus, there are no probative or reductio proofs in relation to analysis as there are in relation to completion. Characteristically, Aristotle conceives of analysis as one sullogismos being transformed into another; he even refers to the process of analysis as a metabasis (51a24-25). Analysis of sullogismoi can occur between any of the figures. Thus, no sullogismos, nor any figure, has preeminence in this respect. There are two rules for analyzing a sullogismos in one figure into another sullogismos in another figure. (1) Analysis is possible only if each sullogismos proves the same problēma — a, e, i, or o (50b5-8); thus, there is no analysis of Barbara, nor of Darii into a second figure sullogismos. (2) Analysis is accomplished using only conversion and premise transposition; thus, neither Baroco nor Bocardo can be analyzed. Aristotle treats the possible analysis of almost every sullogismos and identifies which are not analyzable because the rules are inapplicable.

Analysis is a proof-theoretic process that transforms one sullogismos in any one figure into another sullogismos of any other figure by means of using three conversion rules and premise transposition. Analysis is not concerned with completion nor with eliminating redundant rules. Rather Aristotle aims to promote his students’ facility with syllogistic reasoning. Aristotle’s theorem concerning analysis is that “the sullogismoi in the different figures that prove the same problēma are analyzable into each other [except where per accidens conversion is used]” (see A45, 50b5-7).

6. Concluding remarks. Prior Analytics describes a natural deduction system as part of an underlying logic. It is a proof-theoretic treatise concerned principally to establish and to perfect a deduction system for science. Aristotle knew that deductions about matters pertaining to a given subject matter are content specific and that they employ a topic neutral deduction system; such a system makes evident that given sentences logically follow from other given sentences. One process of deduction is accomplished through taking pairs of given categorical sentences to generate immediate inferences according to prescribed rules, which categorical inferences are then added to the given sentences and then again taken in pairs, to wit, syllogistically, until a final conclusion is obtained (see esp. A25). This process is treated in Prior Analytics in an exactly analogous fashion as chaining immediate inferences by using rules of propositional logic.

We can recognize Aristotle’s keen commitment to logos in respect to his project by what he writes in Prior Analytics A30 about the capability of his syllogistic system.

For if nothing that truly belongs to the subjects has been left out of our collection of facts, then concerning every fact, if a demonstration for it exists, we will be able to find that demonstration and demonstrate it, while if it does not naturally have a demonstration, we will be able to make that evident. (46a24-27)

This passage certainly suggests a modern concern with the completeness of a deduction system, that is, with decidability and whether every logical consequence of a set of sentences is deducible using a given set of deduction rules. Aristotle’s treatment of an ancient logic is strikingly modern respecting its focus on the properties of a deduction system. Aristotle develops just this focus in relation to establishing a formal deduction system in service to truth and apodeiktê epistêmê. In Prior Analytics Aristotle turned his attention specifically to the deduction apparatus used to process information. Once he provided a descriptive model of his system, he could study it and then establish theorems about that system’s rules of deduction. Prior Analytics is a testament to Aristotle’s profound commitment to logos.
References