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There is an ancient puzzle about motion in Plato at *Parmenides* 155e-157b which has been the subject of scholarship by Richard Sorabji and more recently, Nico Strobach.[1] The puzzle, as Plato gives it, can be roughly summarized as follows: At every time, a given object must either be in motion or at rest; there is no third possibility. Also, an object can never be simultaneously both in motion and at rest. The only way for an object to be both in motion and at rest is for it to be in motion and at rest at different times. But how does a thing come to be in motion at one time and at rest at another? It cannot switch at a time when it is in motion. Nor can it switch at a time when it is at rest. This would seem to exhaust the possibilities for the times when the switch could occur. But a thing cannot change without changing.

Plato asks, essentially, when and how an object switches between motion and rest. Aristotle takes up the puzzle in the sixth book of the *Physics (Phys.* 6.3, 234a34-b5), but he recasts it as a problem about what to call an instant separating a period of motion from a period of rest; i.e., shall we call it an instant of motion, an instant of rest, an instant of both motion and rest, or an instant of neither motion nor rest? Saying that the dividing instant is an instant of both motion and rest violates the law of non-contradiction, while saying that it is an instant of neither motion nor rest violates the law of the excluded middle. But saying that it is just an instant of motion or just an instant of rest seems arbitrary, since it bounds both the period of motion and the period of rest.

Aristotle assumes that, *if* one allows motion or rest at an instant, then one must say that an instant dividing periods of motion and rest must be an instant of *both* motion and rest, since "it is the same instant that belongs to both the periods [of motion and of rest]" (*Phys.* 6.3, 234a34). Aristotle argues that, since the law of non-contradiction is non-negotiable, we must reject the assumption that there can be motion or rest at an instant, and this, he implies, avoids violating the law of the excluded middle because "the motion of that which is in motion and the rest of that which is at rest must occupy [a period of] time" (*Phys.* 6.3, 234b8-9). Aristotle's point seems to be that the law of the excluded middle does not apply to motion and rest at instants because they are not the sort of things to exist at instants — motion and rest are defined over periods of time. This allows him to solve the puzzle by saying that the instant dividing periods of motion and rest is an instant of neither motion nor rest, and that this is the case because *all* instants are instants of neither motion nor rest.

It is important to note that while this solution may be fine as far as it goes, it does not constitute a complete answer to Plato's puzzle, which inquires after the status of the instants dividing periods of motion and rest *as well as* what is involved in arriving at and departing from these instants. Now in *Physics* book 3, chapter 1, and book 5, chapter 2, and *Metaphysics* book K, chapter 12, Aristotle recognizes only four kinds of genuine or non-incidental changes, viz., change of quality, change of place, change of size, and generation and destruction,[2] and in *Physics* book 6, chapter 6, he claims that each of these sorts of change is continuous.[3] Since Aristotle commits himself to the view that local motion, as such, is continuous, in order to be consistent, he must describe the acceleration and deceleration involved in coming to and departing from a stand as continuous as well. And indeed, that is what he appears to do in *Physics* book 6, chapter 8, where he argues that there is no first time of coming to a stand. Aristotle argues that since there is no motion at an instant (234a24-b9; 237a14-15), times of motion must be periods of time (239a3-b4). Since times of motion are periods of time and periods of time are infinitely divisible, times of motion must also be infinitely divisible. Since times of coming to a stand are times of motion, times of coming to a stand must be infinitely divisible as well. Hence, if times of coming to a stand are infinitely divisible, there is no first time of coming to a stand. In other words, no matter how small an interval we pick at the start of a period of coming to a stand, there is always a smaller sub-interval within it that is earlier than the interval taken as a whole.

By a similar argument, there should be no last moment of coming to a stand since no matter how small an interval we pick at the end of a period of coming to a stand, there is always a smaller sub-interval within it that is later than the interval taken as a whole. And if within *any* such sub-interval the moving body has a lower velocity than it does within the period of which it is a part, then Aristotle has succeeded in describing continuous coming to a stand.

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[1] Richard Sorabji "Aristotle on the Instant of Change." *Aristotelian Society Proceedings*, Supplementary Volume 50 (1976). See also *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages*. Ithaca: Cornell University Press, 1983, pp. 403-421. Niko Strobach. *The Moment of Change, A Systematic History in the Philosophy of Space and Time*. Dortrecht: Kluwer Academic Publishers, 1998.

[2] *Physics* 3.1, 200b32-201a16, *Physics* 5.2, and *Metaphysics* K 12.

[3] *Physics* 6.6, 237a17-b3 and b9-21.

It is important to recognize just what Aristotle means when he proposes to divide a motion in this way. In *Physics* book 5, chapter 4 he claims that motions are *actually* divided by coming to a stand. Since Aristotle is dividing a period of coming to a stand in *Physics* book 6, chapter 8, we may safely assume that he proposes to divide the motion of coming to a stand *potentially*, but not actually. What this entails, it seems, is to regard the motion as though it were already completed, and then imagine, counterfactually, that it had been sub-divided by a series of pauses. And when we say that the motion of coming to a stand is *infinitely* divisible, what we imagine is not an actually infinite series of pauses, but a finite series of pauses that is infinitely extendable, i.e., that is potentially infinite. This is because a staccato motion, if we actually tried to undertake it, as well as the process of imagining it occurring, are infinitely extendable step-wise processes. This is apparently what Aristotle has in mind in his answer to Zeno's dichotomy paradox, where he claims that the runner traverses an infinite number of half-distances potentially but not actually (*Phys.* 8.8, 263b6-7). We assume that the runner finishes the race, and then consider, counterfactually, the potentially infinite number of places and times that he might have stopped but, in fact, did not.

I claim, however, that on Aristotle's own principles, he is not entitled to assume that the motion of coming to a stand *can* be completed as a prelude to describing it in this way. Recall that Aristotle claims motion is only intelligible over periods of time. If this is the case, then he must conceive of coming to rest in terms a succession of *periods* in which a body is moving at slower and slower velocities. But since velocity may vary continuously, then no matter how slowly a body is moving, it can and must move even more slowly before it comes to rest. As a consequence, in order to come to rest, either a moving body must traverse the whole of an infinite sequence of periods with smaller and smaller velocities, or it must traverse a finite sequence of such periods and then transition to the period of zero velocity discontinuously. Now obviously, recognizing the traversal of an actually infinite series would be a very serious concession for Aristotle because it would violate one of his most basic philosophical commitments — a commitment that finite human minds are up to the task of understanding the universe, because the universe, in its essence, is a finite thing. In Aristotle's view, our ability to understand the world amounts to our ability to comprehend substances or actualities, and we could not do this if the definitions of these things were infinitely complex (*An. Post.* 1.22, 82b37-9). So Aristotle would appear to have no choice but to recognize the discontinuous coming to be and passing away of motion.

The obvious objection at this point would be to claim that my conclusion is not warranted because it is a *potentially* infinite sequence of periods with smaller and smaller velocities that is traversed, and Aristotle countenances the existence of potential infinities in *Physics* book 3 chapters 4-8, as well as their traversal in book 8 chapter 8. There are, however, compelling reasons to reject this suggestion. Aristotle held that continua are ontologically and definitionally prior to their material parts, as an instance of the more general principle that a form/matter composite is ontologically and definitionally prior to its matter *(Metaph.* Z 1). In *Physics* book 4, chapter 9, Aristotle takes a continuous spatial magnitude, for instance, to be a bounded extension, or a form/matter composite consisting of a bounding surface (form) and a spatial extension (matter).[4] And since kinetic and temporal continua are derived from spatial continua, a continuous motion will also stand to its parts as a form/matter composite to its matter.[5]

A consequence of this is that one need not mention the segments of a motion to capture its non-accidental or formal attributes. Only the formal parts of a form/matter composite are salient for definition and understanding (*Metaph*. Z 10, 1035b31 ff.). This allows Aristotle to respond to Zeno's dichotomy paradox by claiming that the runner traverses an infinite number of half-distances, but this does not prevent him from finishing the race, since the racecourse is only accidentally composed of these half-distances. In its essence and form, a race is just a simple traversal of a racecourse (263b6-7).

Now *prima facie*, velocity and any variation in it would appear to be a non-accidental, formal attribute of a motion. And since Aristotle claims that the velocity of a motion is only intelligible over periods of time, then some segmentation of the motion must be presupposed in any account of its variations in velocity. If these variations in velocity are continuous, then an infinite number of segmentations must be presupposed.

But even if one were to deny that variations in velocity are formal attributes of a motion, one cannot deny that its termini are. In *Physics* book 5, Aristotle tells us that motions are individuated by their termini and that they are terminated by coming to a stand. But surely, coming to a stand is just a special sort of variation in velocity, which, on Aristotle's account, must be accounted for by a segmentation of the motion. And if the coming to a stand is continuous, then an infinite segmentation is required. Granted, the approach of Zeno's runner to the finish line need not presuppose the traversal of an infinite sequence of sub-motions, but this is because it is not part of the

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[4] See *Phys.* 4.9, 209b5. Aristotle identifies unbounded extension with matter at 209b9-10.

[5] See *Physics* 4.11.

puzzle that he comes to a stand at the end of the race. If one claims that he *does* come to a stand at the finish line, then an account should be forthcoming as to how he manages to do this.

So again, Aristotle would appear to have no choice but to recognize the discontinuous coming to be and passing away of motion. Indeed, he seems to concede this consequence in *Physics* book 5, chapter 2 (225b33-a6). Here Aristotle claims that there is no process of coming to be (and presumably passing away) of motion, on the grounds that any "change of change and becoming of becoming" involves an infinite regress. Of course, acceleration is a "change of a change," and a less than generous reading of this passage might take it to deny the existence of acceleration altogether, but I am inclined to think that it is only meant to deny acceleration from a stand, insofar as this is equivalent to the coming to be of a motion. Thus, we may speculate that Aristotle was aware that a discontinuous change in velocity at the beginning and the end of a motion was required on the assumption that the traversal of an actual infinity is impossible.

There are two reasons why *we* might be disconcerted with discontinuous changes in velocity, but they would not have bothered Aristotle. First, from a modern perspective, i.e., one that assigns a sense to instantaneous velocity, discontinuous changes in velocity look paradoxical in that it appears as though the decelerating object has two velocities at once. This is not a problem for Aristotle since an object cannot have an instantaneous velocity. The second reason has to do with Newton's Second Law of Motion, which effectively prohibits discontinuous changes in velocity by making force directly proportional to acceleration. If acceleration were infinite, then an infinite force would be required to cause it, but an infinite force is impossible. Aristotle does not have this problem, however, since in his discussion of dynamics in *Physics* book 7, chapter 5 he makes what we would call force (*dunamis*) directly proportional to velocity instead of acceleration (which makes his notion of *dunamis* in a local motion analogous to Newton's definition of "quantity of motion"). For Aristotle, *dunamis* is needed to sustain as well as to initiate motion, and *dunamis* must be continuously applied as the object moves; more *dunamis* if the object is moving faster, less *dunamis* if it is moving slower. But with no concept of inertia, however, change in velocity will be as sudden as the application and the withdrawal of a *dunamis*. And, if the application of *dunamis* is as sudden as the coming in contact of the mover with the thing moved (see *Cael.* 1.11, 280b6-9), then, by this reasoning also, the change in velocity at the beginning and end of a motion will be instantaneous.

In the light of the foregoing, one might be tempted to claim that for Aristotle, it is only the coming to be and passing away of motion that is discontinuous, and that local motion, once it exists, is invariably continuous. Or one might think that if there is a problem, it is a rather minor one that may be classed as an isolated exception to Aristotle's theory. Indeed, Aristotle seems to allow a variety of exceptions to the rule that all genuine changes are continuous.[6] At *De sensu* chapter 6, (446b28-447a6), Aristotle allows that some qualitative changes may, but are not required to occur all at once, as in the simultaneous freezing of all of the parts of a pond. And a few lines earlier (446a18-20), he admits the discontinuity of transitions between colors, tastes, and sounds, due to what he assumes is the limited number of discriminable colors, tastes, and sounds in existence. In *Physics* 7.5, he also allows discontinuity in the force needed to move an object, positing a threshold force, below which a motion will not occur (due to friction, presumably). The example given is ship-hauling, where if it takes 100 men to move a ship 100 feet, it does not follow that one man can move a ship one foot (250a15-19). Finally, in *Metaphysics* book **Z**, chapter 15, and book H, chapters 3 and 5, he claims that substantial forms cannot come into or go out of existence piecemeal, since the unity of a substance is irreducible to the unity of any of its parts (1039b26; 1043b14; 1044b21).

But dismissing, like this, the problem I have identified, overlooks the fact that it is perfectly generalizable to transitions between any two velocities. Suppose that a moving body traveling at 6 miles per hour decelerates to a velocity of 5 miles per hour. If the number of decelerations that the body undergoes is finite, how will the body ultimately decelerate to 5 miles per hour without undergoing a discontinuous change in velocity? No matter how close the velocity gets to 5 miles per hour (e.g., 5.1, 5.01, 5.001, 5.0001 m.p.h., etc.), if the number of decelerations is finite, then the ultimate deceleration to 5 miles per hour must be discontinuous. If passing to and from a period of zero velocity is discontinuous because Aristotle cannot account for how a continuous transition between motion to rest ultimately takes place, so too must the passage to and from any period with some positive velocity. If this is so, then non-uniform motions will not only discontinuous at their beginnings and endings, but throughout.

This, I think, highlights a basic shortcoming of Aristotle's concept of motion. The idea that motion is simply the traversal from place A at time t to place B at time $t + 1$ makes it impossible for Aristotle to conceive of continuous changes in velocity. For Aristotle, the velocity associated with the traversal of an object between spatial

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[6] Changes that fall outside of the category of genuine change may be discontinuous as well, e.g., relational changes. Things may stand or fail to stand in relations to other things, but there is no process of coming to stand in a relation, or coming not to stand in a relation (*Phys.* 225b11-13; 246b11-12; 247b4). But what is at issue in Plato's puzzle is the genuine change of local motion.

termini is always a *non-instantaneous* velocity, and a non-instantaneous velocity always masks the variation in the velocity over the period of change. Aristotle can divide the motion up as finely as he likes, but unless he embraces the concept of instantaneous velocity, there will always be jumps or drops in velocity as the object moves from one motion subsegment to the next.