Binghamton University

The Open Repository @ Binghamton (The ORB)

The Society for Ancient Greek Philosophy Newsletter

12-28-2007

Studying Mathematics for the Sake of the Good

Andrew Payne

St. Joseph's University, apayne@sju.edu

Follow this and additional works at: <https://orb.binghamton.edu/sagp>

Part of the [Ancient History, Greek and Roman through Late Antiquity Commons](#), [Ancient Philosophy Commons](#), and the [History of Philosophy Commons](#)

Recommended Citation

Payne, Andrew, "Studying Mathematics for the Sake of the Good" (2007). *The Society for Ancient Greek Philosophy Newsletter*. 405.

<https://orb.binghamton.edu/sagp/405>

This Article is brought to you for free and open access by The Open Repository @ Binghamton (The ORB). It has been accepted for inclusion in The Society for Ancient Greek Philosophy Newsletter by an authorized administrator of The Open Repository @ Binghamton (The ORB). For more information, please contact ORB@binghamton.edu.

Studying Mathematics for the Sake of the Good

Andrew Payne, *St. Joseph's University*

In the *Republic*, Socrates describes the good as the end of all human action: “Every soul pursues the good and does what it does for its sake. It divines that the good is something but it is perplexed and cannot adequately grasp what it is or acquire the sort of stable beliefs it has about other things, and so it misses the benefits, if any, that even those other things may give.”¹ I wish to examine how humans act for the sake of the good in the sections of the *Republic* following this passage. Human action is oriented toward the good in several distinct ways, one of which is illustrated by the procedures of mathematics in pursuing knowledge: without intending to achieve insight into forms such as the square itself, we can act for the sake of achieving this end.

The most familiar way to describe action for the sake of the good involves what I will call agency-centered teleology. It tells us that an action is for the sake of an end only if the agent has an awareness of the end as his goal and acts in a way that shows an awareness of how his actions promote that goal.² All human beings desire what is good, and they pursue it differently on the basis of their greater or lesser cognitive grasp of the good. An action is explained by referring to a desire for some end and a belief which represents that goal as attainable by some action.³ The discussion of mathematics in Books VI and VII of the *Republic* deserves our attention because it reveals a different way of acting for the sake of an end. The prospective guardians of Socrates’ best city study mathematics in order to achieve an understanding of the good, but they do so without intending or believing that this study will lead them to this understanding. Their study of mathematics produces understanding of the good, but this is an unforeseen and unintended end. As we will see, this way of performing an action for the sake of an end that is not intended is illustrated more concretely by a passage from the image of the line, where mathematicians are said to make arguments about visible images for the sake of understanding a form such as the square itself.

A quick examination of the pages following the claim at 505d-e that all humans act for the good reveals three different ways in which we act for the sake of the good. Some pursue the good from the mistaken belief that it consists in pleasure or knowledge or some other lesser good. These people act for the sake of the good by pursuing what seems falsely to them to be the good. They would not accept pleasure or knowledge if they were aware that it only seemed to be the good but in reality was not the good itself.⁴

¹ *Republic* 505d11-e4. All translations from the *Republic*, unless otherwise noted, are by Grube.

² The terminology and underlying concepts used here to describe agency-centered teleology are taken from André Ariew, “Platonic and Aristotelian Roots of Teleological Arguments,” in *Functions: New Essays in the Philosophy of Psychology and Biology* ed. André Ariew, Robert Cummins, and Mark Perlman (New York: Oxford, 2002), 8-10.

³ This strategy for explaining action goes back to Aristotle and is present in contemporary philosophy of action inspired by Donald Davidson. Actions are explained by citing the reasons for which they are performed, and these reasons consist in a belief-desire pair, of which the desire provides the goal and the belief describes how the goal may be attained. Alfred Mele provides the following account of a reason for which an action is done: “Any reason for which a person acts has a conative element that motivates the pursuit of a goal or a subgoal and a representational element that constitutes a plan, or an element of a plan, for achieving the pertinent goal or subgoal.” See Mele, *The Springs of Action* (New York: Oxford University Press, 1992), 110.

⁴ Rep. 505b-e.

A second way of seeking the good involves possessing true beliefs about the good, a possibility Socrates mentions when Adeimantus asks him to state what he believes concerning the good. Finally there is a third and best way of pursuing the good, in which we act for the sake of the good on the basis of full insight into the form of the good. This best state is required of the guardians in the best city, who act for the sake of the form of the good by gaining knowledge of the form and then using this knowledge as a paradigm for their own conduct: “And once they’ve seen the good itself, they must each in turn put the city, its citizens, and themselves in order, using it as their model.”⁵

These three modes of acting for the sake of the good are examples of agency-centered teleology: they involve the universal desire for the good conjoined with either false belief, true belief, or knowledge about the good. However, this is not a complete specification of the various ways in which humans act for the sake of the good. The guardians-to-be, those potential philosopher-rulers who are still making the ascent toward the good but have not yet reached it, must count as instances of the general truth that all human beings act for the sake of the good. Yet their manner of acting for the sake of the good is not captured by any of these three modes of acting for the sake of the good. The ascent toward the good is accomplished by extensive training in various mathematical *technai*. These fields of study do not qualify as knowledge in the strict sense because they do not examine and account for their basic principles and starting points, the hypotheses mentioned at 510b5–c6 and 533c1, but still they provide access to true belief rather than falsehood. Nor can these actions be performed on the basis of knowledge about the good. This knowledge is achieved only at the summit of the ascent, and our concern is to explain how the philosopher-rulers act for the good before gaining this knowledge.

Several passages indicate that it is not in virtue of their true beliefs that the young guardians act for the sake of the good. Socrates disparages the value of true belief when Adeimantus asks that he share his beliefs about the good: “Haven’t you noticed that beliefs without knowledge are shameful and ugly things? The best of them are blind – or do you think that those who express a true belief without understanding are any different from blind people who happen to travel the right road?”⁶ If the guardians in training have only true belief about the good, they will lack an account of the cause of the various good things they enjoy. They will lack an understanding of the form of the good, since belief concerns the many just or good things but not the one form of justice or the form of the good (479d-e). Since they will lack understanding of the form of the good, any success they have in reaching this overarching cause of all that exists will be a matter of luck, like blind men who happen by chance to find the right road to their destination. If this is how the guardians act for the sake of the good in their education, then their ability to reach their end would be purely a matter of chance. More than this is needed for the guardians’ training and for their action to be oriented toward the good.

These observations direct us toward the task of specifying the cognitive state of the guardians by virtue of which they act for the sake of the good as they are educated toward the good. The cognitive state in question must fall short of knowledge of the good, since that is the end that is not yet attained during this education, but be higher than true belief, which is oriented toward the good only by chance. Socrates’s ranking of the different intellectual powers in the image of the line suggests thought or *dianoia* as the

⁵ Rep. 540a-b.

⁶ 506c6-9; Grube’s translation amended.

logical candidate, since it is said to be between belief and understanding (511d4-5). *Dianoia* is the faculty by which we come to know the mathematical sciences, and Socrates prescribes an intensive training in mathematics for the guardians as a prelude to knowing the good. Mathematical study contributes to understanding the form of the good by directing the soul away from the visible realm and towards the intelligible: “Then it [geometry] drags the soul towards truth and produces philosophic thought by directing upwards what now we wrongly direct downwards” (527b9-11; see also 523a, 525a-c, 526b). This feature of mathematical study within the education of the guardians is uncontroversial; less clear is its connection with acting for the sake of the good. In what sense do the guardians engage in these mathematical studies for the sake of the good, if mathematics studies such value-neutral entities as numbers and plane figures?

Much of our understanding of *dianoia* must rely on 510b2-511b2, in which Socrates discusses the third section of the divided line. The third section of the line, or the first section of the intelligible realm, contains the objects grasped by *dianoia* while the fourth contains those understood by dialectic. The third section is of special concern for us, since the guardians in training will employ *dianoia* as they study the objects contained in this section.⁷ We learn that *dianoia* is compelled to start from hypotheses which mathematicians lay down as things known, that *dianoia* employs images, and that it does not ascend to a nonhypothetical first principle. Concerning the mathematicians’ use of images, Socrates speaks as follows:

...[A]lthough they use visible figures and make claims about them, their thought isn’t directed to them but to those other things that they are like. They make their claims for the sake of the square itself and the diagonal itself, not the diagonal they draw, and similarly with the others. These figures that they make and draw, of which shadows and reflections in water are images, they now in turn use as images, in seeking to see those others themselves that one cannot see except by means of *dianoia*.

Mathematicians employ visible objects and make claims about them while thinking about other objects, apparently forms such as the square itself, to which the visible objects bear a likeness. The use of visible figures or diagrams is an exercise of *dianoia* carried out for the sake of a form, which is the phenomenon that interests us.

⁷ I wish to avoid commitment on the much-discussed issue of what exactly these objects are. Some commentators identify these contents as forms, either strictly mathematical forms such as the square itself or a more extensive range of forms: A. S. Ferguson, “Plato’s Simile of Light,” *Classical Quarterly* 15, no. 4 (1921), 131-52; R. Hackforth, “Plato’s Divided Line and Dialectic,” *Classical Quarterly* 36, no. 1 (1942), 1-9, and Richard Robinson, *Plato’s Earlier Dialectic* (New York: Oxford University Press, 1953), 192-20. Others restrict the contents of the third section to the so-called mathematical, the plurality of abstract squares or diagonals which mathematicians cognize. See J. Adam, *The Republic of Plato* (Cambridge: Cambridge University Press, 1902), vol. II, Appendix I to Book VI and Anders Wedberg, *Plato’s Philosophy of Mathematics* (Stockholm: Almqvist & Wiksell, 1955), 99-111. On the first reading, the difference between the third and the fourth sections of the line, and between the realms discerned by *dianoia* and understanding respectively, is a difference of intellectual method but not of objects. On the second, a distinct class of objects as well as a distinct method corresponds to each section of the line. Plato does not describe the line in sufficient detail for us to resolve the question of whether the third section of the line contains its own class of objects. This indicates that he is concerned more with the ultimate result and end of *dianoia*, namely understanding mathematical forms, than with the issue of whether the understanding is obtained directly or is mediated by grasp of the mathematical. In either case, we must face the question of how mathematical inquiry occurs for the sake of insight into mathematical forms.

As a first step towards explicating the notion of making a claim about a visible object for the sake of a form, we can rephrase this as a matter of making a claim about a visible object for the sake of a conceptual grasp of the form. With regards to a form, what we can achieve by our study is not the existence of the form or any modification of its condition. As a form, it exists independently of those who think of it. The relevant product of mathematical inquiry that can qualify as that for the sake of which the inquiry is pursued is the student's own comprehension of the form.

How does working with visible figures help the student of geometry to gain an apprehension of a form? Geometers posit various figures as their hypotheses, so we can suppose that they posit the existence of squares, though they do not think it necessary to say what it is to be a square (510c2-d1). Working with an intuitive sense of what a square is and what properties it possesses (all four sides of a square are of equal length) they draw diagrams of squares and use these images to prove various results about the squares they draw. By inspection of a diagram of a square and one of its diagonals, the mathematicians can convince themselves that a square can be divided into two equal triangles. Already by this point in a "proof" of a trivial conclusion, our mathematicians are thinking of entities other than the ones they draw and see: the two equal triangles are not the same as the two triangles drawn in the diagram, which are most likely not fully equal. The square which the two equal triangles constitute will also not be the same as the one that is drawn. This acquaintance with a square that is not seen but is known by *dianoia* can then stimulate more focused reflection on what makes this thing a square and what it is to be a square. Success in answering these questions will lead to some apprehension, direct or indirect, clear or obscure, of the square itself. So *dianoia* performs the crucial task of turning the soul from the visible to the intelligible realm.⁸

However, we still must explain how visible squares are studied for the sake of understanding the square itself. It is not enough merely to assert that examining one sort of object leads to or causes understanding of another; if I study mathematical diagrams by drawing them on a blackboard with chalk, this may cause me to learn about the properties of chalk, but I did not study mathematical diagrams for the sake of insight into chalk. It cannot be the case that mathematicians study diagrams for the sake of gaining insight into the square itself by virtue of intending to gain insight into the square itself. This insight into the square itself is the result of mathematical investigation, and not one of its preconditions. As Socrates describes the procedures of the mathematicians, they begin by positing squares and other figures as hypotheses, but they neglect to give an account of what these figures are. The grasp of these hypotheses does not amount as insight into the forms. It is only after examining diagrammed squares and making claims about them gaining insight into the square itself is recognized as a task by the students of mathematicians. Because mathematicians lack awareness of forms as they begin their inquiry, we cannot treat their studying diagrams for the sake of understanding forms as a case of agency-centered teleology.⁹

⁸ This presentation of mathematical study leading to insight into the square itself follows Michael Morgan, "Belief, Knowledge, and Learning in Plato's Middle Dialogues," *Canadian Journal of Philosophy* Supplementary Volume IX (1987), 63-100, and Ian Mueller, "Mathematical Method and Philosophical Truth," in *The Cambridge Companion to Plato*, ed. Richard Kraut (New York: Cambridge University Press, 1992), 170-99.

⁹ Typically commentators discuss 510d7-8 and the notion of studying images for the sake of understanding forms as a case of setting out with the desire or intention of learning about the forms. John Brentlinger says

An explanation of how mathematicians study diagrams for the sake of understanding forms will draw upon several causal and normative relations between studying diagrams and understanding forms. First, the acts of studying diagrams must help cause or produce the understanding of forms as a result. Second, this result must be of greater value than its cause; to do x for the sake of y implies that, in this context at least, y is more worth having than x . Finally, that this understanding of forms results must be a criterion of success in studying diagrams. Studying chalk diagrams may produce knowledge of chalk, but this result is no criterion of success for mathematical study. To summarize formally these three conditions, suppose that I is a field of intellectual inquiry and F is some form. Then the following three conditions describe formally the relation that holds between the particular actions of studying diagrams and understanding the square itself, such that the former are performed for the sake of the latter:

C_1 Studying I produces understanding of F

C_2 Understanding F is of greater value than studying I

C_3 If studying I is done in the best way, then it produces understanding of F ¹⁰

These conditions are satisfied by the description of mathematical inquiry at 510b-511b. The process of making claims about visible squares plays a causal role in producing understanding of the square itself. This understanding is more valuable than the process of working with diagrams. The third condition summarizes Socrates's views on the limitations of mathematical practice and how these limitations can be overcome. He believes that reliance on hypotheses which are not themselves known through an account places limits on the status of mathematics as knowledge. The claims that mathematicians make rest on unexamined assumptions that squares exist as certain figures with a list of properties (internal angles of 90 degrees, all sides equal, etc.). With the knowledge of the square itself, this qualification can be lifted. A mathematician who knows what a square is and how this is connected to a square's essential properties will derive many of the same conclusions about squares as does the mathematician who relies only on hypotheses, but the former's conclusions will enjoy a higher epistemic status. The form-knowing mathematician has "destroyed the hypotheses" (533c8) by replacing them in the structure of his investigations with knowledge of the square itself.¹¹ Without insight into the square itself, this non-hypothetical practice of mathematics is not possible. As a result, a necessary condition for making arguments about squares in the best way is achieving insight into the square itself.

of this passage that it "unequivocally asserts that the dianoietic sciences seek to know the forms..."; see Brentlinger, "The Divided Line and Plato's Theory of Intermediates," *Phronesis* 8 (1963), 149. The passage does assert unequivocally that forms such as the square itself and the diagonal itself are what mathematical sciences allow us to think about. But the sense in which these sciences *seek* those forms is equivocal. Mathematicians do not seek the forms by setting out to learn about them; they seek them rather in the sense that the understanding of forms, once gained, upholds the claims to knowledge that mathematicians make.

¹⁰ This description of the relation between studying mathematics and achieving understanding of forms was inspired by reflection on Richard Kraut's description of the for-the-sake-of relation in Book I of Aristotle's *Nicomachean Ethics*. See Kraut, *Aristotle on the Human Good* (Princeton, New Jersey: Princeton University Press, 1989), 200-1.

¹¹ This account of the limitations of typical mathematical practice and their remedy by knowledge of forms draws upon Richard Hare, "Plato and the Mathematicians" in *New Essays on Plato and Aristotle*, ed. Renford Bambrough (New York: Humanities Press, 1965), 21-38.

This suggests that mathematicians study visible squares for the sake of the square itself in the following way. They begin with hypotheses about squares and other figures that lack the status of knowledge. As they use visible squares to solve particular problems and to draw conclusions about squares, they do not anticipate or plan to gain insight into the square itself. They do, however, claim to know the entities they hypothesize as well as the conclusions they draw from study of visible squares – they posit their hypotheses “as if they knew them” (510c6). Insight into the square itself is an unintended result of this mathematical inquiry, a result that allows mathematicians to fulfill their epistemic ambitions. Achieving this result flows from the proper pursuit of their inquiry and is the crucial step needed for mathematicians to make good on their claims to knowledge. In this sense, their use of images and study of visible diagrams is carried out for the sake of an end they do not intend, namely insight into forms.

Our original question was the following: How do the guardians-to-be act for the sake of the good? Now we can answer this question in light of our account of how mathematicians study diagrammed squares for the sake of understanding the square itself. As was mentioned, Socrates requires his prospective guardians to complete an extended course of mathematical studies. His rationale for this is that mathematics drags the soul away from the visible realm and toward the intelligible, a process that culminates in the vision of the form of the good. This result produced by mathematical study is surely of greater value than the act of studying itself. And achieving this result is a criterion of success in mathematical study. As Socrates says of harmonics pursued in the proper way, “it’s useful in the search for the beautiful and the good. But pursued for any other purpose, it’s useless” (531c6-7). With knowledge of the good the guardians can understand all else as flowing from a nonhypothetical first principle, and so vindicate their claims to mathematical knowledge. It is in this sense that the guardians exercise *dianoia* for the sake of the good; they need not intend to gain knowledge of the good.