

Table of Contents

PREFACE v

PART I: MATHEMATICAL INTRODUCTION TO STATE SPACE

1. Models of Linear Systems 3

- 1.1 Linear Systems and State Equations 3
 - 1.1.1 Definitions and Review 4
 - 1.1.2 Physical System Modeling 6
 - Example 1.1: Mechanical System Equations 10*
 - Example 1.2: Electrical System Equations 11*
 - 1.1.3 State Equations 13
 - Example 1.3: State Variables for the Mechanical System 15*
 - Example 1.4: State Variables for the Electrical System 15*
 - 1.1.4 Discrete-Time Systems 23
 - Example 1.5: Direct Difference Equation Modeling of a Savings Account 24*
 - Example 1.6: A Difference Equation for a Predator-Prey System 25*
 - Example 1.7: Discretized Differential Equations for a Harmonic Oscillator 26*
 - 1.1.5 Relationship to Transfer Functions 27
- 1.2 Linearization of Nonlinear Equations 29
 - 1.2.1 Linearizing Functions 29
 - 1.2.2 Linearizing Differential Equations 32
 - Example 1.8: Linearization of a Differential Equation for an Inverted Pendulum 33*
- 1.3 Summary 36
- 1.4 Problems 37
- 1.5 References and Further Reading 44

Listings in *italics* denote applications-oriented examples.

2. Vectors and Vector Spaces 45

- 2.1 Vectors 46
 - 2.1.1 Familiar Euclidean Vectors 46
- 2.2 Vector Spaces 48
 - 2.2.1 Fields 48
 - Example 2.1: Candidate Fields 49
 - Example 2.2: Candidate Vector Spaces 50
 - 2.2.2 Linear Dependence and Independence 52
 - Example 2.3: Linear Dependence of Vectors 53
 - Example 2.4: Vectors of Rational Polynomials 54
 - 2.2.3 Bases 57
 - Example 2.5: Spaces and Their Bases 60
 - Example 2.6: *Common Infinite-Dimensional Spaces* 61
 - 2.2.4 Change of Basis 62
 - Example 2.7: Change of Basis 64
 - 2.2.5 Rank and Degeneracy 66
 - 2.2.6 Inner Products 67
 - Example 2.8: Inner Product for a Function Space 68
 - 2.2.7 Norms 68
 - Example 2.9: Norms 69
 - 2.2.8 Some Other Terms 70
- 2.3 Gram-Schmidt Orthonormalization 71
- 2.4 Subspaces and the Projection Theorem 75
 - 2.4.1 Subspaces 75
 - 2.4.2 The Projection Theorem 76
 - Example 2.10: Projection of a Vector 77
 - Example 2.11: *Finite Fourier Series* 79
- 2.5 Linear Algebras 83
- 2.6 Summary 84
- 2.7 Problems 85
- 2.8 References and Further Reading 91

3. Linear Operators on Vector Spaces 93

- 3.1 Definition of a Linear Operator 94
 - 3.1.1 Range and Null Spaces 94
 - Example 3.1: The Range and Null Space of a Projection 95
 - 3.1.2 Matrix Representations of Linear Operators 97
 - Example 3.2: *Rotation Matrices* 100
 - Example 3.3: *Composite Rotations: Pitch, Roll, and Yaw* 102
 - Example 3.4: Matrix Representation of the Projection Operator 104
 - Example 3.5: *Projection onto an Arbitrary Plane* 105
 - Example 3.6: *Computer Graphics and Object Rotation* 106
 - 3.1.3 Linear Operators in Different Bases 108
 - Example 3.7: Matrix Operators and Change of Basis 109
- 3.2 Operators as Spaces 114

- 3.2.1 Operator Norms 115
 - 3.2.2 Adjoint Operators 116
 - 3.3 Simultaneous Linear Equations 116
 - 3.3.1 Multiple and Nonexistent Solutions 118
 - Example 3.8: Unique and Nonexistent Solutions 122
 - Example 3.9: Pseudoinverse Solution 126
 - Example 3.10: Controllability of Discrete-Time Systems 127
 - Example 3.11: Observability of Discrete-Time Systems 128
 - 3.3.2 Least-Squared Error Solutions and Experimental Data Analysis 130
 - Example 3.12: Analyzing Data All at Once 131
 - Example 3.13: Recursive Least-Squares 135
 - 3.4 Summary 136
 - 3.5 Problems 137
 - 3.6 References and Further Reading 145
4. Eigenvalues and Eigenvectors 147
- 4.1 A-Invariant Subspaces 147
 - 4.2 Definitions of Eigenvectors and Eigenvalues 148
 - Example 4.1: Electric Fields 149
 - 4.3 Finding Eigenvalues and Eigenvectors 150
 - Example 4.2: Simple Eigenvalue Problem 150
 - Example 4.3: Simple Eigenvector Problem 151
 - Example 4.4: Complex Eigenvalues and Eigenvectors 152
 - Example 4.5: Eigenvalues and Eigenvectors of Operators on Function Spaces 153
 - 4.4 The Basis of Eigenvectors 155
 - 4.4.1 Changing to the Basis of Eigenvectors 155
 - Example 4.6: Diagonalization of Operators 156
 - 4.4.2 Repeated Eigenvalues 157
 - 4.4.3 Generalized Eigenvectors 159
 - Example 4.7: A Small Number of Generalized Eigenvectors 160
 - Example 4.8: Top-Down Method for a Small Matrix 164
 - Example 4.9: Multiple Chains of Generalized Eigenvectors 165
 - 4.4.4 When n Independent Eigenvectors Exist 167
 - Example 4.10: Moments of Inertia 171
 - 4.4.5 Jordan Canonical Forms 172
 - Example 4.11: Jordan Forms for Some Previous Examples 173
 - 4.5 Singular Values 175
 - 4.5.1 Properties of Singular Value Decompositions 176
 - Example 4.12: Singular Value Decomposition 176
 - 4.5.2 Some Applications of Singular Values 177
 - Example 4.13: Singular Value Decompositions in Robotics 181
 - 4.6 Summary 186
 - 4.7 Problems 187
 - 4.8 References and Further Reading 192

5. Functions of Vectors and Matrices 195

- 5.1 Linear Functionals 195
 - 5.1.1 Changing the Basis of a Functional 196
- 5.2 Multilinear Functionals 197
 - 5.2.1 Changing the Basis of a Bilinear Form 198
 - 5.2.2 Bilinear Forms as Inner Products 199
- 5.3 Quadratic Forms 200
 - 5.3.1 Testing for Definiteness 201
 - 5.3.2 Changing the Basis of a Quadratic Form 202
 - 5.3.3 Geometry of Quadratic Forms 203
 - Example 5.1: Statistical Error Analysis 204*
 - Example 5.2: Conic Sections 206*
- 5.4 Functions of Matrices 208
 - 5.4.1 Cayley-Hamilton Theorem 209
 - Example 5.3: Application of the Cayley-Hamilton Theorem to Matrix Inverses 210
 - 5.4.2 Using the Cayley-Hamilton Theorem to Compute Matrix Functions 211
 - Example 5.4: Closed-Form Matrix Functions 212
 - Example 5.5: Matrix Functions with Repeated Eigenvalues 214
 - Example 5.6: Matrix Exponential of Jordan Form 218
 - 5.4.3 Minimal Polynomials 219
 - Example 5.7: Minimal Polynomials for Jordan Forms 220
- 5.5 Summary 220
- 5.6 Problems 221
- 5.7 References and Further Reading 225

PART II: ANALYSIS AND CONTROL OF STATE SPACE SYSTEMS**6. Solutions to State Equations 229**

- 6.1 Linear, Time-Invariant (LTI) Systems 229
 - Example 6.1: Simple LTI System 231*
- 6.2 Homogeneous Systems 232
 - 6.2.1 Phase Portraits 233
- 6.3 System Modes and Decompositions 245
 - 6.3.1 A Phase Portrait Revisited 249
 - Example 6.2: Sketching a Phase Portrait Using Qualitative Analysis 250
- 6.4 The Time-Varying Case 252
 - 6.4.1 State Fundamental Solution Matrix 252
 - 6.4.2 The State-Transition Matrix 254

	Example 6.3: State-Transition Matrix Using Series Expansion	255
6.5	Solving Discrete-Time Systems	257
6.5.1	Discretization	258
6.5.2	Discrete-Time State-Transition Matrix	259
6.5.3	Time-Invariant Discrete-Time Systems	260
	Example 6.4: Discretization of a System	261
6.6	Summary	262
6.7	Problems	264
6.8	References and Further Reading	267
<u>7.</u>	<u>System Stability</u>	<u>269</u>
7.1	Lyapunov Stability	269
7.1.1	Equilibrium Points	269
	Example 7.1: Equilibrium Points for Linear and Nonlinear Systems	270
7.1.2	Classification of Equilibria	274
7.1.3	Testing For Lyapunov Stability	274
7.1.4	Eigenvalues of Time-Varying Systems	279
	Example 7.2: An Unstable System with Negative Eigenvalues	279
7.2	Lyapunov's Direct Method	280
7.2.1	Lyapunov Functions and Testing	282
	Example 7.3: Lyapunov Function for a Nonlinear System	284
7.2.2	Lyapunov Functions for LTI Systems	285
	Example 7.4: Stability Test for a Parameterized System	289
	Example 7.5: Discrete-Time LTI Lyapunov Equation	290
7.2.3	Unstable Systems	291
	Example 7.6: An Unstable System	291
7.3	External Stability	292
7.3.1	Bounded Input, Bounded Output Stability	292
7.3.2	BIBO Stability for Time-Invariant Systems	297
7.4	Relationship Between Stability Types	299
	Example 7.7: A BIBO, but Not Asymptotically Stable System	299
7.5	Summary	301
7.6	Problems	302
7.7	References and Further Reading	308
<u>8.</u>	<u>Controllability and Observability</u>	<u>311</u>
8.1	Definitions	311
8.2	Controllability Tests for LTI Systems	312
8.2.1	The Fundamental Tests for Controllability and Observability	312
	Example 8.1: Controllability and Observability of a Circuit	315

- 8.2.2 Popov-Belevitch-Hautus Tests 319
- 8.2.3 Controllability and Observability of Jordan Forms 321
 - Example 8.2: Controllability and Observability of Multivariable Jordan Forms 324
- 8.2.4 Controllable and Observable Canonical Forms 326
- 8.2.5 Similarity Transformations and Controllability 330
- 8.3 Modal Controllability and Observability 331
 - 8.3.1 Geometric Interpretation of Modal Controllability 331
 - 8.3.2 Kalman Decompositions 334
 - Example 8.3: Decomposition of an Uncontrollable System 336
 - 8.3.3 Stabilizability and Detectability 338
- 8.4 Controllability and Observability of Time-Varying Systems 339
 - 8.4.1 Continuous-Time Systems 339
 - Example 8.4: Controllability for a Time-Varying System 344
 - 8.4.2 Reachability and Reconstructibility 347
- 8.5 Discrete-Time Systems 348
 - 8.5.1 Controllability and Reachability 348
 - Example 8.5: A Discrete-Time Control Sequence 351
 - 8.5.2 Observability and Reconstructibility 353
- 8.6 Controllability and Observability Under Sampling 357
- 8.7 Summary 357
- 8.8 Problems 359
- 8.9 References and Further Reading 365

9. System Realizations 367

- 9.1 Minimal Realizations 368
- 9.2 Specific Realizations 372
 - 9.2.1 Controllable Canonical Realization 373
 - 9.2.2 Observable Canonical Realization 374
 - 9.2.3 Jordan Canonical Realizations 375
- 9.3 Markov Parameters 378
 - 9.3.1 Proof of the Minimal Realization Theorem 380
 - 9.3.2 Hankel Matrices and System Order 383
 - Example 9.1: Realization from a Hankel Matrix 385
- 9.4 Balanced Realizations 387
 - 9.4.1 Grammians and Signal Energy 388
 - 9.4.2 Internal Balancing 390
- 9.5 Discrete-Time System Identification 393
 - 9.5.1 Eigensystem Realization Algorithm 394
 - 9.5.2 Simplified ERA Identification 396
- 9.6 Summary 397
- 9.7 Problems 399
- 9.8 References and Further Reading 403

10. State Feedback and Observers 405

- 10.1 State Feedback for SISO Systems 406
 - 10.1.1 Choosing Gain Matrices 409
 - Example 10.1: SISO State Feedback 410
 - 10.1.2 Properties of State Feedback 418
 - Example 10.2: State Feedback for an Uncontrollable System 419
- 10.2 Multivariable Canonical Forms and Feedback 421
 - 10.2.1 Controllability Indices and Canonical Forms 422
 - Example 10.3: Multi-input Pole Placement 426
- 10.3 Observers 430
 - 10.3.1 Continuous-Time Observers 431
 - Example 10.4: An Observer and Controller With Simulation 437
 - 10.3.2 Discrete-Time Observers 442
 - 10.3.3 Output Feedback and Functional Observers 445
- 10.4 Summary 446
- 10.5 Problems 448
- 10.6 References and Further Reading 454

11. Introduction to Optimal Control and Estimation 457

- 11.1 The Principle of Optimality 458
 - 11.1.1 Discrete-Time LQ Control 460
 - Example 11.1: A Discrete-Time LQ Controller 464
 - 11.1.2 Continuous-Time LQ Control 465
 - 11.1.3 Infinite-Horizon Control 473
 - Example 11.2: Continuous-Time LQ Control 477
- 11.2 Optimal Estimators 481
 - 11.2.1 Models of Systems with Noise 482
 - 11.2.2 The Discrete-Time Kalman Filter 483
 - 11.2.3 The Continuous-Time Kalman Filter 491
 - 11.2.4 Properties of Kalman Filters 495
 - Optimality of Kalman Filters 495
 - Steady-State Behavior 496
 - Example 11.3: A Continuous-Time Kalman Filter 497
- 11.3 LQG Control 500
 - Example 11.4: LQG Controller for Disturbance Rejection 501
- 11.4 Summary 505
- 11.5 Problems 507
- 11.6 References and Further Reading 513

A. Mathematical Tables and Identities 515

- Basic Matrix Operations 515
- Transposes 517
- Determinants 517
- Inverses 518

Trace 521
Special Matrices and Matrix Forms 522
Matrix Decompositions 523
Matrix Calculus 524
Integrals 527

B. MATLAB Command Summaries 529

Command Reference 530
References 558

INDEX 561